

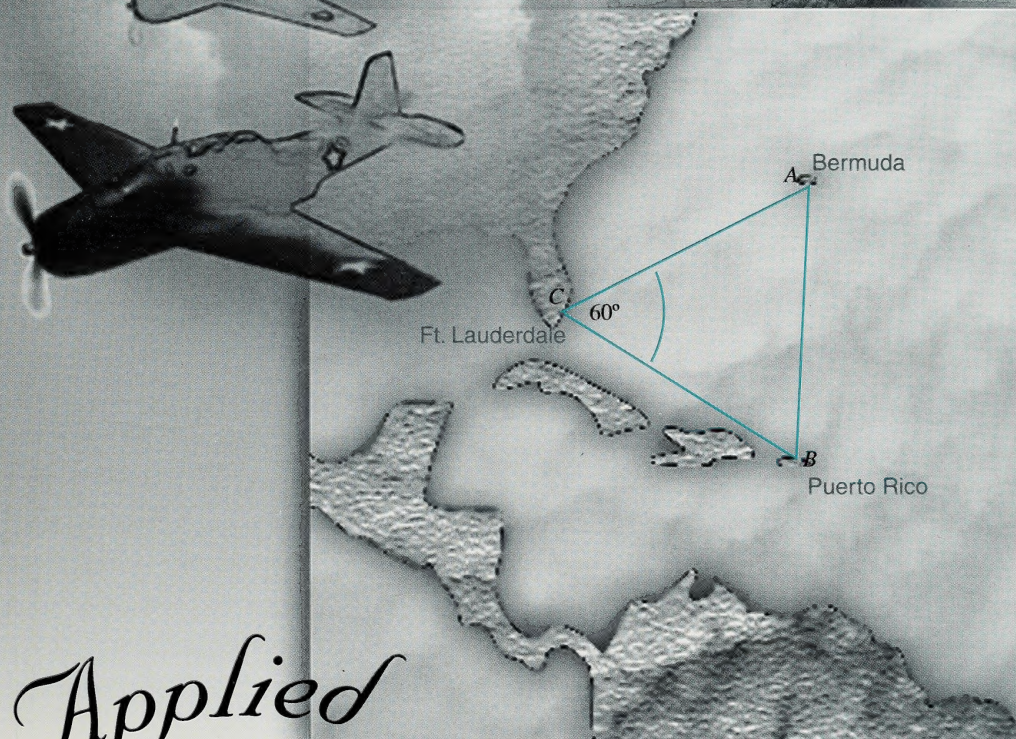
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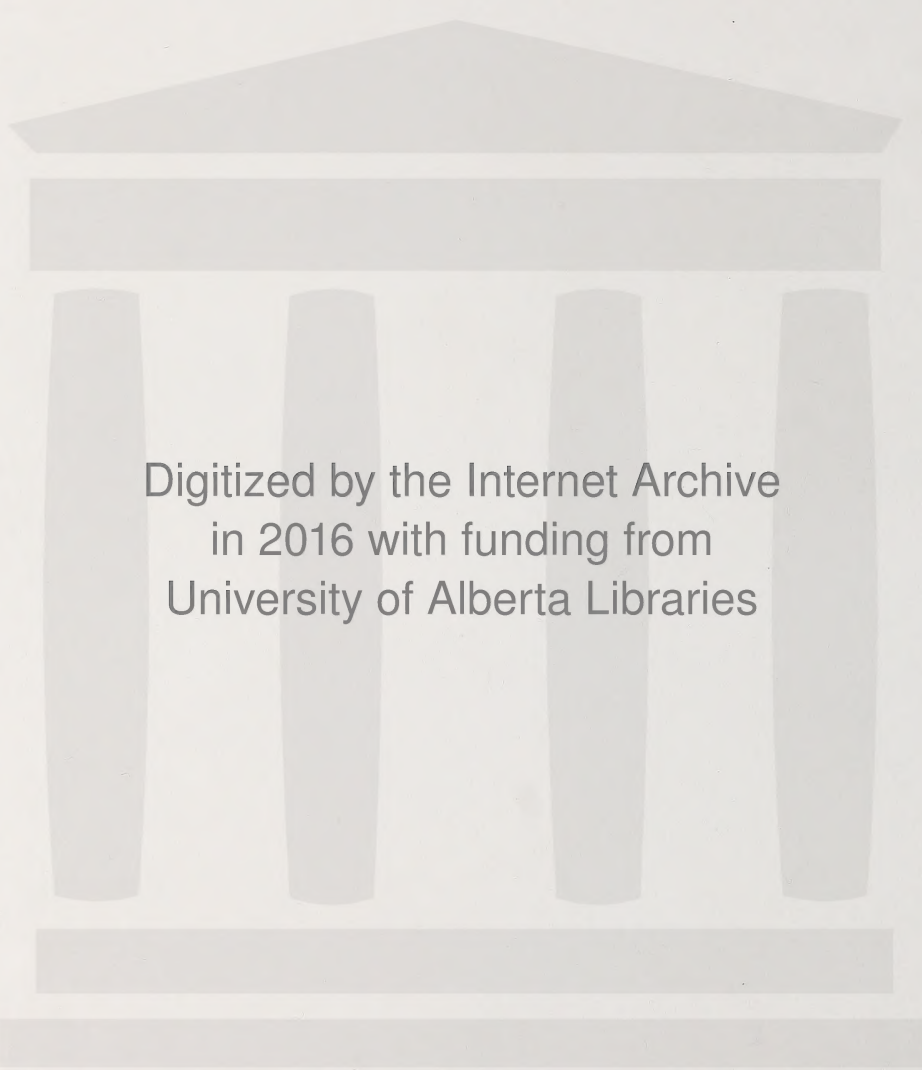
Mathematics 10

LINE SEGMENTS



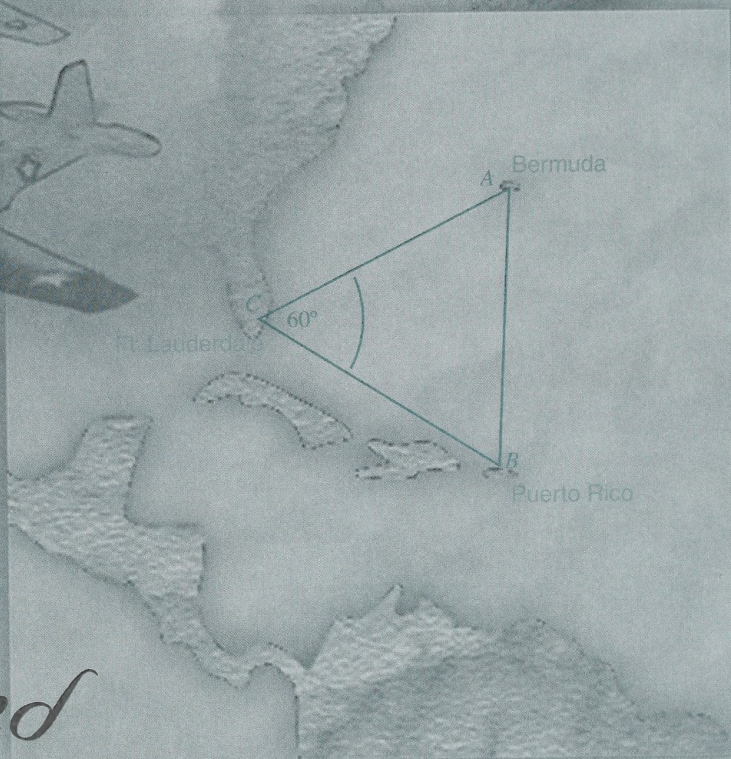
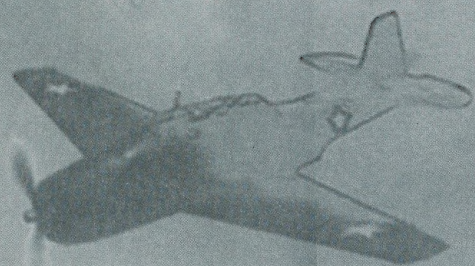
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Module 5



Applied

Mathematics 10

LINE SEGMENTS

Applied Mathematics 10
Student Module Booklet
Module 5
Line Segments
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	



The Learning Technologies Branch has an Internet site that you may find useful.
The address is as follows:

<http://www.learning.gov.ab.ca/lrb>

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

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Welcome

Applied Mathematics 10

Module 1
Measurement

Module 2
Number Patterns in Tables

Module 3
Relations and Functions

Module 4
Sampling

Module 5
Line Segments

Module 6
Linear Functions

Module 7
Trigonometry

*Welcome
to Module 5.
We hope you'll
enjoy your study
of Line Segments.*



Applied Mathematics 10 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

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*Submit the
Module Project.*



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*Submit the
Module Assignment.*

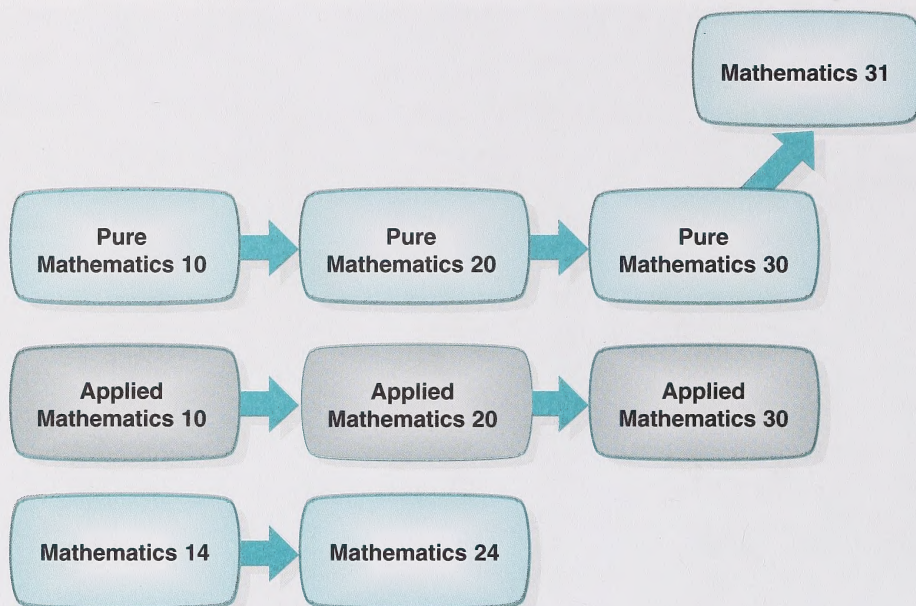


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Introduction to Applied Mathematics 10

Applied Mathematics 10 is the first course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different post-secondary and career plans.

You may find it helpful to read mathematics updates on Alberta Learning's website:

<http://www.learning.gov.ab.ca/studentprograms>

Before enrolling in Applied Mathematics 10, it is recommended that you talk with a school counsellor about your career plans.



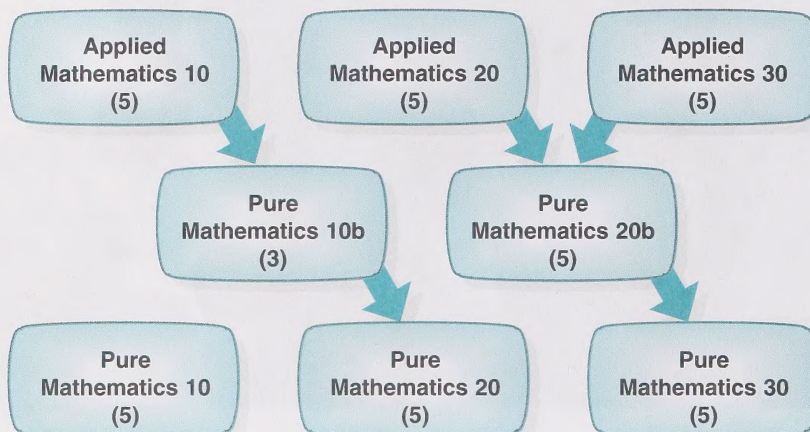
TRANSFERRING FROM THE APPLIED MATHEMATICS PROGRAM

You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are specific to either the applied or pure mathematics courses.

The following table shows some of the common topics and some of the independent topics in the mathematics program.

Applied Topics	Common Topics	Pure Topics
<ul style="list-style-type: none"> • linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems 	<ul style="list-style-type: none"> • spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve 	<ul style="list-style-type: none"> • irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics. If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may take the 3-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20 or Applied Mathematics 30, you may take the 5-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.

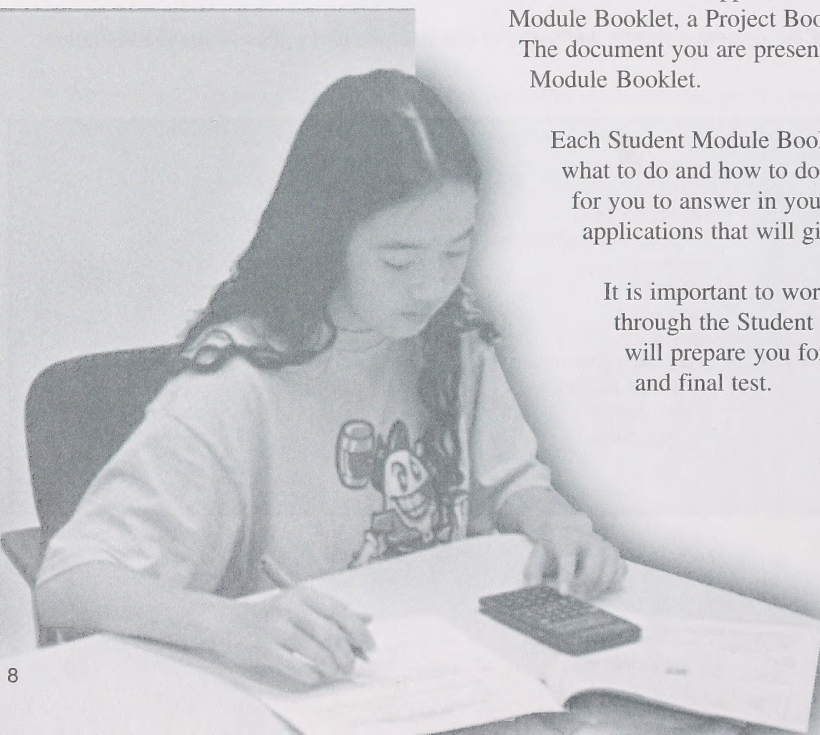


STRATEGIES FOR COMPLETING APPLIED MATHEMATICS 10

For each module in Applied Mathematics 10, there is a Student Module Booklet, a Project Booklet, and an Assignment Booklet. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the projects, assignments, and final test.



Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also, store your marked assignments here.
- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.
- Keep a section of your binder to record new skills and concepts as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words and recording useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.
- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an “aha!” experience) or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 10 to be your study partner. You will find that having a friend with whom you can discuss mathematical ideas will make your studying more enjoyable.



Resources You Will Need

In addition to the distance learning materials for Applied Mathematics 10, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 10 Source Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (1999)
- a binder, lined loose-leaf paper, graph paper, dividers, a pencil, and an eraser
- metric and imperial measuring devices, such as a ruler, a tape measure, a yardstick, a vernier caliper, and a micrometer
- a mathematical instrument set (compass, protractor, and triangles)
- a computer installed with a spreadsheet program

Note: Two popular spreadsheet programs are *ClarisWorks™* and *Microsoft® Excel*. The examples in this course show *Microsoft® Excel*.

- a graphing calculator

Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Brands	Texas Instruments	Sharp	Casio
Models	TI-82 TI-83 TI-83 Plus TI-86 TI-89 TI-92 TI-92 Plus	EL-9600c EL-9600* EL-9300* EL-9200*	CFX-9850Ga-Plus CFX-9850G* CFX-9800G* FX-9700 series*

*no longer commercially available, but may be available on loan from your school division

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you view the video *The TI-83 Graphing Calculator Video Tutor* to discover some of the calculator's features.

Many of the resources you will need for this course may be purchased from the Learning Resources Distributing Centre (LRDC). Following is the LRDC website:

<http://www.lrdc.edc.gov.ab.ca>

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

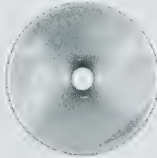
Visual Cues

You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the explanations given to discover what the various icons prompt you to do.

- Refer to the textbook.



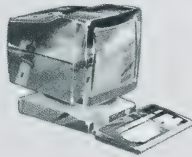
- Use the companion CD for Applied Mathematics 10.



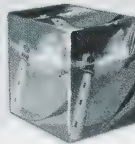
- Use mathematical instruments, measuring devices, and other materials.



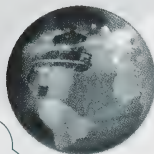
- Work with a computer.



- Complete the module project or assignment.



- Explore the Internet.



Remember: Any Internet website address given in this module is subject to change.

MODULE OVERVIEW

Facts About Cycling Race Tracks

Bicycle track races are always run in a counterclockwise direction, so all turns are to the left. The turns on a velodrome are usually “banked,” or constructed at an angle. There is no standard for velodrome length—tracks vary in length from 100 m to 500 m (or more) with 250 m, 333.3 m, and 400 m being the most common.

The velodrome at the Juan de Fuca Recreation Centre in Victoria, British Columbia, has a 333.3-m cycling track with turns banked at 28° and straightaways banked at 10° .

The Superdrome™ in Frisco, Texas, has a 250-m track with turns banked at 44° and straightaways banked at 13° .

You can visit the following website to take a virtual tour of the Superdrome™ in Frisco, Texas.

<http://www.superdrome.com/about.html>

Have you ever watched a bicycle race in a velodrome? In some of the track races, riders jockey for position as they try to outwit each other before the final dash to the finish line.

A cycle race in a velodrome takes place on a banked track. The area inside the track is the “infield”—it is used as a warm-up area for riders and as a staging area for competitors, coaches, and officials. Encircling the infield at the inner edge of the track is a flat, paved surface called the “apron.” The apron can be used as a warm-up area, a place for mounting and dismounting bikes, or as a run-off area for exiting the track in case of an accident.

Painted lines running around the full oval offer measurement information and safe haven for riders under racing and training situations. The black line indicates the inside of the track. The wide, light blue band below the black line is the transition between the track proper and the inside run-off area. Farther up from the black line is a red line, which signifies where a rider cannot pass inside another rider in front of him or her. Sprinters often ride just above this line to tempt the rider behind into coming underneath. They then move down below the red line shutting off the route through. Once a rider has entered below the red line within the last 200 m, the rider must stay there.

In this module, you will explore line segments. You will work with the coordinate system and develop formulas to find the length and the midpoint of line segments, given their endpoints. You will also develop a formula to find the slope of line segments that have been plotted on a grid.



EVALUATION

Accompanying this Student Module Booklet is a Project Booklet and an Assignment Booklet. Your grading in this module will be based upon the module project and the module assignment you submit for evaluation. The mark distribution is as follows:

Module Project	50 marks
Module Assignment	50 marks

TOTAL 100 marks

Remember that Activities 1 to 4 in this Student Module Booklet will prepare you for completing the module project and the module assignment. Work through these activities carefully and compare your answers with the suggested answers in the Appendix.

The Follow-up Activities provide extra help and enrichment. You may choose to do some or all the questions in the Follow-up Activities. Again, you should compare your answers with the suggested answers provided in the Appendix.

MODULE PROJECT

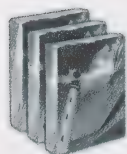
DESIGNING A WATER RIDE

NATHAN BENNICORRIS

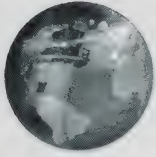
Beginning the Project

Have you ever been on a water ride? Many families today enjoy these rides, and water parks exist in most regions in Canada. Water slides are so popular that they can even be found in hotels and shopping centres.

Your project for Module 5: Line Segments is called Designing a Water Ride. You will research water rides, do an investigation on water rides, and collect data from the investigation. You will describe your design for a water ride and make a sketch of the ride. You will make a scale drawing on graph paper of the side view (elevation profile). You will label the coordinates of the endpoints of each section of the water ride. You will also indicate the slope of each section of the water ride on the side view.



Turn to page 198 of the textbook, read “Designing a Water Ride,” and begin answering the questions posed. Store your responses in the project section of your mathematics binder.

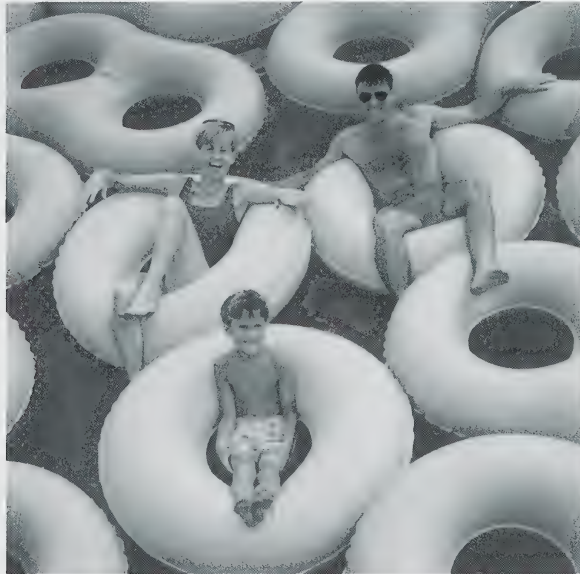


In order to answer all the questions, you will need to do some research. To begin your research, you may wish to visit the following sites on the Internet and view photographs of various water rides:

- <http://www.proslide.com/wride.htm>
- <http://www.amusementleisure.com/WaterRides.htm>

You may also visit the Internet site for Addison Wesley Longman Ltd., as stated on page 199 of the textbook. This site has links to several sites you may find helpful.

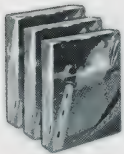
In your research, you will notice that the designs for water rides vary. On some rides, the slides are enclosed; on others, the slides are open. Some slides are curved; other slides are straight. Moreover, the participants may sit on mats, inner tubes, or rafts.



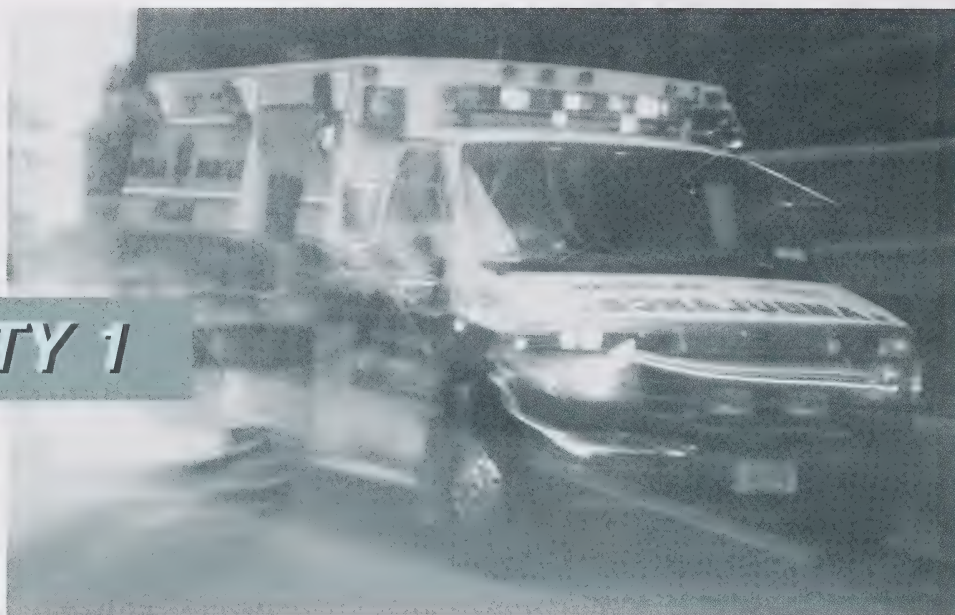
You may be able to visit and enjoy a water ride. Experiencing a water ride may assist you in designing your project.

Activities 1 to 4 and the follow-up activities in this module will prepare you for this project. As you work through the activities, continue to research design ideas and respond to the questions posed on page 198 of the textbook.

You will be given more directions on how to complete this project later in the module.



ACTIVITY 1



Distance

Numbers are used in the everyday world to identify people, places, and things. For example, Wayne Gretzky is known as Number 99 (the number he wore on his jersey), the prime minister of Canada lives at 22 Sussex Drive, Ottawa, and the telephone number 911 is used in cases of emergency.

A **coordinate** system of latitude and longitude is used to indicate the location of places on the surface of Earth.



Following are some examples of coordinate points.

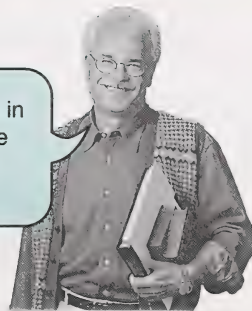
City	Geographic Coordinates
Victoria, British Columbia	48° N latitude and 123° W longitude
Edmonton, Alberta	53° N latitude and 113° W longitude
Ottawa, Ontario	45° N latitude and 75° W longitude
Halifax, Nova Scotia	44° N latitude and 63° W longitude

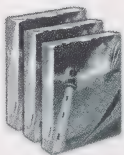
1. Use the preceding chart to answer parts a. and b.
 - a. How many degrees latitude farther north is Edmonton than Halifax?
 - b. How many degrees longitude farther west is Victoria than Ottawa?



Compare your responses with the suggested answers in the Appendix, Activity 1, page 44.

A coordinate system is used in mathematics to indicate the placement of points, line segments, and curves.





Turn to pages 202 and 203 in your textbook and complete exercises 1 and 2 of “Investigation 2: The Distance between Two Points.” Then answer the following questions.

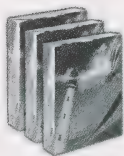
2.
 - a. Find the length of the vertical line segment AC by counting the number of squares between A and C .
 - b. How can you find the length of AC using the y -coordinates of A and C ?
 - c. Generalize a rule for finding the length of a vertical line segment, given the coordinates of its endpoints.
3.
 - a. Find the length of the horizontal line segment BC by counting the number of squares between B and C .
 - b. How can you find the length of BC using the x -coordinates of B and C ?
 - c. Generalize a rule for finding the length of a horizontal line segment, given the coordinates of its endpoints.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 44–45.



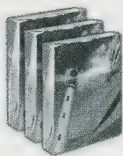
You will now discover how to find the length of an **oblique** line segment.

An oblique line segment is at a slant to the x - and y -axis.



4. Turn to page 203 of the textbook and answer exercises 4 and 5 of “Investigation 2: The Distance between Two Points.” Round your answer to one decimal place.
Note: If you need to review the Pythagorean Theorem, turn to the Student Reference on page 428 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 1, page 45.



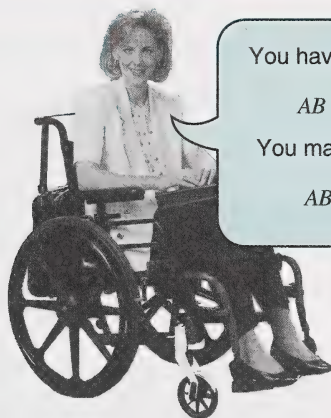
Your work in Investigation 2 has prepared you for the **distance formula**.



Turn to pages 203 to 205 of the textbook and read the information that begins below “Investigation 2: The Distance between Two Points” and continues on page 204. Then work through “Example 1: Calculate the distance between two points using the distance formula.”

5. Turn to page 207 of the textbook and answer exercises 3 and 4 of “Discussing the Ideas.”

Compare your responses with the suggested answers in the Appendix, Activity 1, page 45.



You have discovered the distance formula

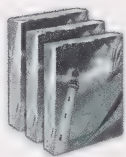
$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}.$$

You may also see this formula written as

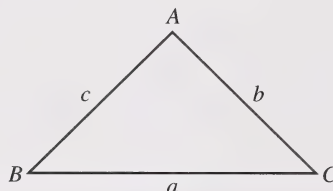
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This formula may look scary, but it simply means that you complete the following steps:

- Step 1:** Subtract the x -coordinate of the first endpoint from the x -coordinate of the second endpoint. Then, square the difference.
- Step 2:** Subtract the y -coordinate of the first endpoint from the y -coordinate of the second endpoint. Then, square the difference.
- Step 3:** Add the squares from Step 1 and Step 2. Then, find the positive square root of the sum.



6. Turn to pages 208 to 210 of the textbook and answer exercises 3.a., 4, 5, 6, and 7 of “Exercises: Checking Your Skills.”
7. Turn to page 210 of the textbook and answer exercise 8 of “Exercises: Extending Your Thinking.” **Note:** The side opposite $\angle A$ is a , the side opposite $\angle B$ is b , and the side opposite $\angle C$ is c .



Compare your responses with the suggested answers in the Appendix, Activity 1, pages 45–51.

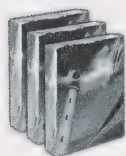
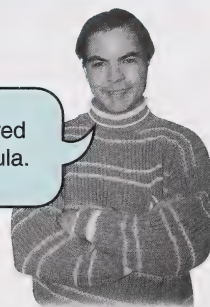
You may wish to find out more about Heron of Alexandria. Use the Internet to do a search of “Heron of Alexandria” or “mathematicians+Heron.” One site you may find of interest is as follows:

<http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Heron.html>



LOOKING BACK

In this activity, you discovered and used the distance formula.



8. Turn to page 212 of the textbook and answer “Communicating the Ideas.”

Compare your responses with the suggested answers in the Appendix, Activity 1, page 51.

ACTIVITY 2



Midpoints

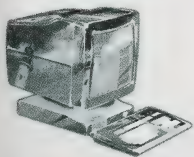
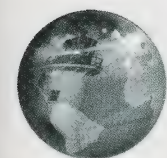
Almost any artist will tell you that mathematics is needed to create paintings or sculptures. Artists need to know about geometry, ratio and proportion, and symmetry.

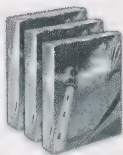
One artist that is particularly well known for using mathematics in his artwork is M. C. Escher; you may wish to do a search for Escher on the Internet. You can also use the search term *tessellation*. If you do not have access to the Internet, you should be able to find information on Escher in any library.

In this activity, you will use the coordinate system to find the **midpoints** of line segments. The midpoint of a line segment is the point that divides the segment into two equal parts.

Turn to pages 238 to 240 of your textbook and do **one** of the following:

- If you have access to a computer with *Geometer's Sketchpad*® or similar investigative geometry software, complete exercises 1 to 15 of "Investigation 2: Midpoint of a Line Segment Using Technology."
- If you do not have access to a computer with *Geometer's Sketchpad*® or similar investigative geometry software, complete exercises 1 to 12 of "Investigation 1: Midpoint of a Line Segment Using a Coordinate Grid."





Turn to page 240 of the textbook and read the information following Investigation 2.

1. Turn to page 242 of the textbook and answer exercise 2 of “Discussing the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 2 , page 52.

You have discovered a method of finding the midpoint of a line segment using its endpoints. If the endpoints of the line segment AB are (x_A, y_A) and (x_B, y_B) , the coordinates of the midpoint of line segment AB are $\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$.



Turn to pages 243 and 244 of the textbook and work through “Example: Use the midpoint of a line segment.”

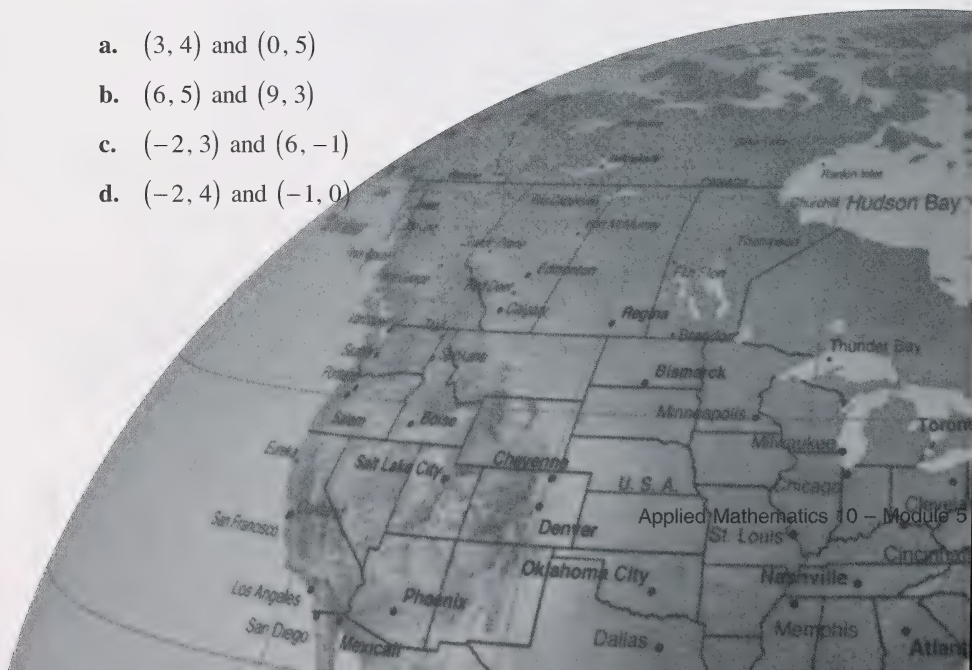
Note: When using your graphing calculator to determine the midpoint, remember the parentheses are needed to show that each term in the numerator is divided by the denominator.

In other words, to find x_M in the example, press the following key sequence:

(4 • 5 + 1 3 • 3) ÷ 2

2. Find the midpoint of line segments with the following endpoints:

- a. $(3, 4)$ and $(0, 5)$
- b. $(6, 5)$ and $(9, 3)$
- c. $(-2, 3)$ and $(6, -1)$
- d. $(-2, 4)$ and $(-1, 0)$





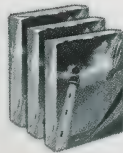
3. Find the centre of a circle that has $A(0, 3)$ and $B(3, -4)$ as the endpoints of the diameter.
4. Turn to page 244 of the textbook and answer exercises 3 and 4 of “Exercises: Checking Your Skills.”



Compare your responses with the suggested answers in the Appendix, Activity 2, pages 52–54.

LOOKING BACK

In this activity, you found the midpoints of line segments.



5. Turn to page 245 of the textbook and answer “Communicating the Ideas.”

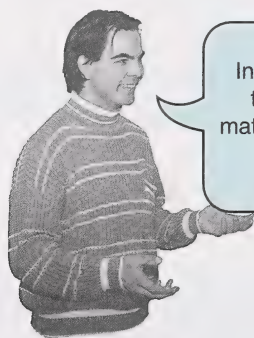
Compare your responses with the suggested answers in the Appendix, Activity 2, page 55.

ACTIVITY 3



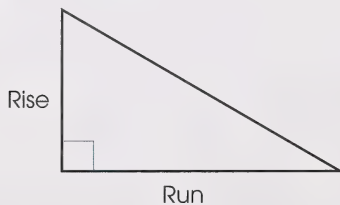
Slope

Do you enjoy skiing or snowboarding on mountain slopes? For safety reasons, the trails at most ski resorts are clearly marked according to their degree of difficulty. The degree of difficulty may be influenced by the steepness of the slope.



In this activity, you will investigate the **slope** of a line segment. In mathematics, slope is the ratio of the **rise** to the **run**.

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

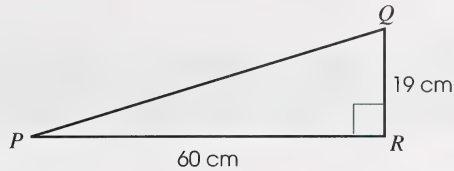


Work through the following example.

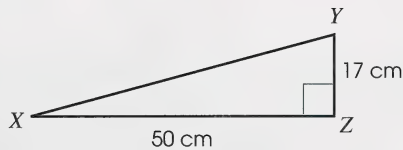
Example

Compare the slopes of the following ramps. Which ramp is steeper?

Ramp 1



Ramp 2



Solution

Step 1: Determine the slope of each ramp. Round to two decimal places.

Ramp 1

$$\begin{aligned}\text{Slope} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{19}{60} \\ &\doteq 0.32\end{aligned}$$

The slope of Ramp 1 is about 0.32.

Ramp 2

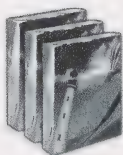
$$\begin{aligned}\text{Slope} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{17}{50} \\ &= 0.34\end{aligned}$$

The slope of Ramp 2 is about 0.34.

Step 2: Compare the slopes.

$$0.34 > 0.32$$

Ramp 2 has the steeper slope.



1. Turn to page 224 of the textbook and answer exercise 1 of “Exercises: Checking Your Skills.”
2. For safety, the slope of a ladder should be 4. If the top of a ladder is leaned up against a house, at a point 2.5 m from the ground, how far from the base of the house should the ladder be placed?

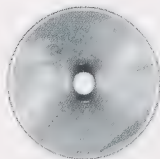
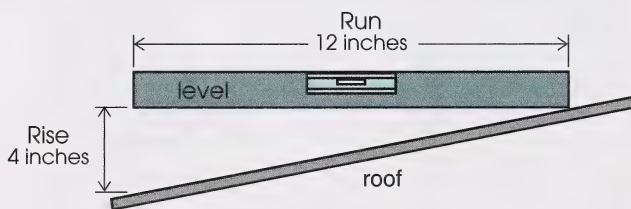
Compare your responses with the suggested answers in the Appendix, Activity 3, page 55.

The **pitch** of a roof is related to its slope. The pitch of a roof is determined by the vertical rise in inches for every horizontal run of 12 inches.

A fairly easy way to determine the pitch of a roof is by using a level. Set one end of a 12-inch level (a spirit level or carpenter’s level) on the roof surface. Make the level horizontal. That is, centre the bubble in the spirit. Then, take a tape measure or ruler and find the distance from the free end of the level down to the roof surface.



This method is illustrated in the following diagram. The roof has a 4-inch rise for the 12-inch run. Therefore, its pitch is expressed as “4 in 12” or “4 on 12.” **Note:** A pitch of 4 in 12 has a slope of $\frac{4}{12} = \frac{1}{3}$.



View the multimedia segment entitled “Roof Pitch” on the companion CD.

3. Explain why the slopes of roofs in cold regions are generally steeper than the slopes of roofs in warm regions.

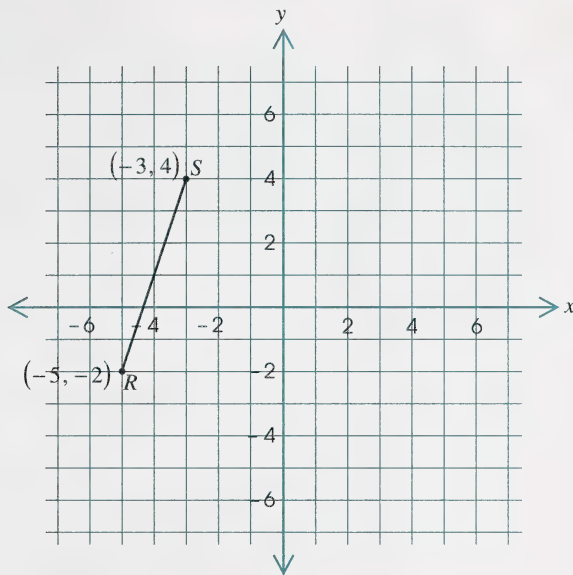
Compare your response with the suggested answer in the Appendix, Activity 3, page 55.

Now that you are familiar with the concept of slope in the real world, you will explore the concept of slope on a coordinate plane.

Work through the following examples.

Example

Find the slope of line segment RS .



Solution

Step 1: To find the values for the rise and run, consider the magnitude (size) and direction of the change vertically and horizontally.

To get from R to S , you can go up (in a positive direction) 6 units, and then you must go right (in a positive direction) 2 units.

Therefore, the rise is $+6$ and the run is $+2$.

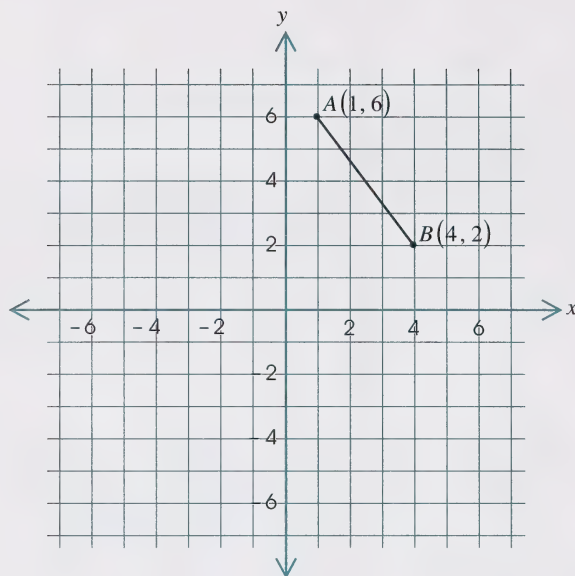
Step 2: Calculate the slope.

$$\begin{aligned}\text{Slope} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{+6}{+2} \\ &= 3\end{aligned}$$

The slope of line segment RS is 3.

Example

Find the slope of line segment AB .



Solution

Step 1: To find the values for the rise and run, consider the magnitude (size) and direction of the change vertically and horizontally.

To get from A to B , you can go down (in a negative direction) 4 units, and then you must go right (in a positive direction) 3 units.

Therefore, the rise is -4 and the run is $+3$.

Step 2: Calculate the slope.

$$\begin{aligned}\text{Slope} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{-4}{+3} \\ &= -\frac{4}{3}\end{aligned}$$

The slope of line segment AB is $-\frac{4}{3}$.

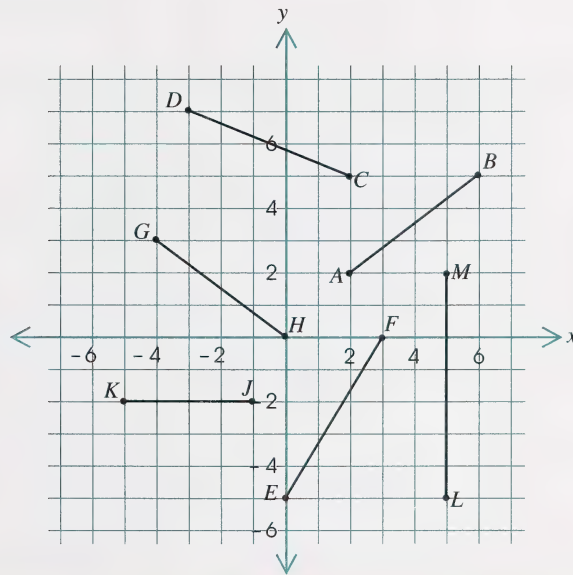


What do you notice about the slopes of line segments on the coordinate plane as compared to the slopes of structures in the real world?

Slopes of structures in the real world are always positive. Slopes of line segments on the coordinate plane may be positive or negative.



4. Calculate the slope of each line segment in the following coordinate plane.



Compare your responses with the suggested answers in the Appendix, Activity 3, pages 55–56.

Recall the formulas you have used in this module when dealing with a line segment AB that has endpoints $A(x_A, y_A)$ and $B(x_B, y_B)$:

- the length of $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$
- the midpoint of $AB = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right)$

A similar formula can be developed for the slope of line segment AB .

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

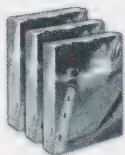
$$= \frac{y_B - y_A}{x_B - x_A}$$

Subtract the y -coordinate of the first endpoint from the y -coordinate of the second endpoint.

Subtract the x -coordinate of the first endpoint from the x -coordinate of the second endpoint.

If you prefer, you may use numerical subscripts in the formula.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$



Turn to pages 220 and 221 of the textbook and work through “Example 1: Calculate the slope of line segments in the coordinate plane.”

5. Turn to page 224 of the textbook and answer exercises 3 and 4 of “Discussing the Ideas.”
6. Turn to pages 224 and 225 of the textbook and answer exercises 2 and 4 of “Exercises: Checking Your Skills.”
7. Turn to page 228 of the textbook and answer exercise 10 of “Exercises: Checking Your Skills.” In exercise 10.a., express each slope as a fraction.
8. Turn to page 228 of the textbook and determine the coordinates of each point in exercise 11 of “Exercises: Extending Your Thinking.” (You do not have to find the length of each section.)

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 56–63.

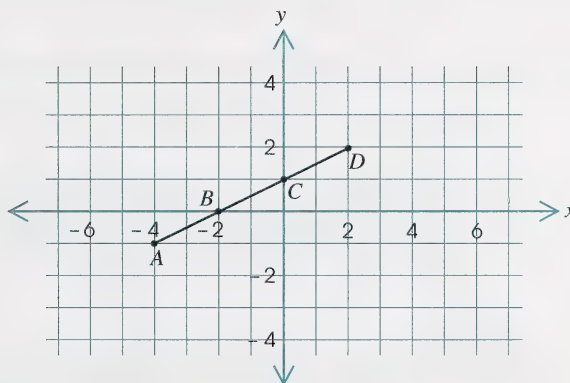


If the slopes of the line segments joining any pair of a series of points are the same, the points are **collinear**. That is, the points lie on the same line segment.

Work through the following example.

Example

Points $A(-4, -1)$, $B(-2, 0)$, $C(0, 1)$, and $D(2, 2)$ are on a coordinate plane. Are points A , B , C , and D collinear? Explain why or why not.



Solution

If the slopes of the line segments joining any pair of the series of points are the same, the points are collinear.

Step 1: Determine the slope of AB .

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0 - (-1)}{-2 - (-4)} \\ &= \frac{0 + 1}{-2 + 4} \\ &= \frac{1}{2}\end{aligned}$$

$$A(-4, -1)$$

$$B(-2, 0)$$

$$x_1 = -4; x_2 = -2$$

$$y_1 = -1; y_2 = 0$$

Step 2: Determine the slope of BC .

$$\begin{aligned}\text{Slope} &= \frac{1-0}{0-(-2)} \\ &= \frac{1}{0+2} \\ &= \frac{1}{2}\end{aligned}$$

$$B(-2, 0)$$

$$C(0, 1)$$

$$x_1 = -2; x_2 = 0$$

$$y_1 = 0; y_2 = 1$$

Step 3: Determine the slope of CD .

$$\begin{aligned}\text{Slope} &= \frac{2-1}{2-0} \\ &= \frac{1}{2}\end{aligned}$$

$$C(0, 1)$$

$$D(2, 2)$$

$$x_1 = 0; x_2 = 2$$

$$y_1 = 1; y_2 = 2$$

Step 4: Compare the slopes. The slopes are equal. The points are collinear.

9. Are the points $P(-4, 6)$, $Q(-1, 2)$, and $R(2, -2)$ collinear?

Compare your responses with the suggested answers in the Appendix, Activity 3, page 64.

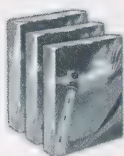
LOOKING BACK



In this activity, you worked with slope.

10. Turn to page 229 of the textbook and answer “Communicating the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 3, page 65.



ACTIVITY 4



Parallel and Perpendicular Line Segments

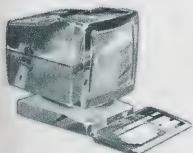
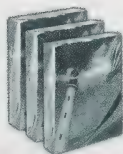
Have you ever noticed the parallel and perpendicular line segments that are evident in the skeleton of a building that is being constructed?

In this activity, you will examine the slopes of parallel and perpendicular line segments.



Turn to pages 232 to 234 of your textbook and do **one** of the following:

- If you have access to a computer with *Geometer's Sketchpad*® or similar investigative geometry software, complete exercises 1 to 18 of "Investigation 3: Slopes of Parallel and Perpendicular Lines Using Technology."
- If you do not have access to a computer with *Geometer's Sketchpad*® or similar investigative geometry software, complete exercises 1 to 6 of "Investigation 1: Slopes of Parallel Lines." Then, turn to page 233 of the textbook and complete exercises 1 to 7 of "Investigation 2: Slopes of Perpendicular Line Segments."



1. What did you discover about the slopes of parallel line segments?
2. What did you discover about the slopes of perpendicular line segments?

Compare your responses with the suggested answers in the Appendix, Activity 4, page 65.

The slope of a line segment is often identified by the variable m .



Find the multimedia segment entitled “Slope” on the companion CD.

Notice that there are two line segments. The slope, m , of the purple line segment (as well as the reciprocal, $\frac{1}{m}$) is written in purple in the top box on the right side of the screen. Similarly, the slope, m , of the orange line segment is printed in orange in the second box on the right side of the screen. In addition, each slope is described in words.



You can slide or rotate each line segment by clicking on an endpoint and dragging. Experiment with moving the line segments and discovering how the slope of each line segment changes.

Here are some suggestions:

- Try placing the segment one on top of the other. What happens in the boxes at the right?
- Try sliding one segment so that segments are parallel. What happens in the boxes at the right?
- Try rotating one segment until the two segments are perpendicular. What happens in the boxes on the right?

You may wish to make some notes summarizing what you discover. A good way to display your findings is in a chart or table.



3. Turn to page 236 of the textbook and answer exercises 1 to 3 of “Discussing the Ideas.”
4. Two line segments are perpendicular if the product of their slopes is -1 . When is this statement not true? Explain.

Compare your responses with the suggested answers in the Appendix, Activity 4, page 65.

Next, turn to pages 235 and 236 in the textbook and work through “Example: Determine whether line segments are perpendicular.”

5. Turn to pages 236 and 237 of the textbook and answer exercises 1 to 3 of “Exercises: Checking Your Skills.” **Note:** For textbook exercise 3, use the slopes to determine the pairs of line segments that are perpendicular.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 66–69.

LOOKING BACK



In this activity, you explored the slopes of parallel line segments and perpendicular line segments.

6. Turn to page 237 of the textbook and answer “Communicating the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 4, page 69.



Follow-up Activities



This module dealt with Chapter 5: Line Segments in the textbook.

Turn to page 248 of the textbook and review the skills and concepts that were developed in this module, as well as the important results and formulas that you discovered.

Turn to page 250 of the textbook and do the following.

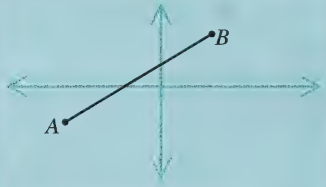
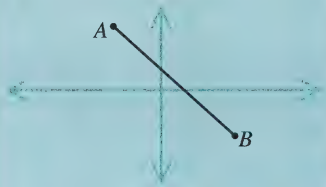
1. Answer exercise 8 of Part B of “What Should I Be Able To Do?”
2. Answer exercises 9 and 10 of Part B of “What Should I Be Able To Do?”

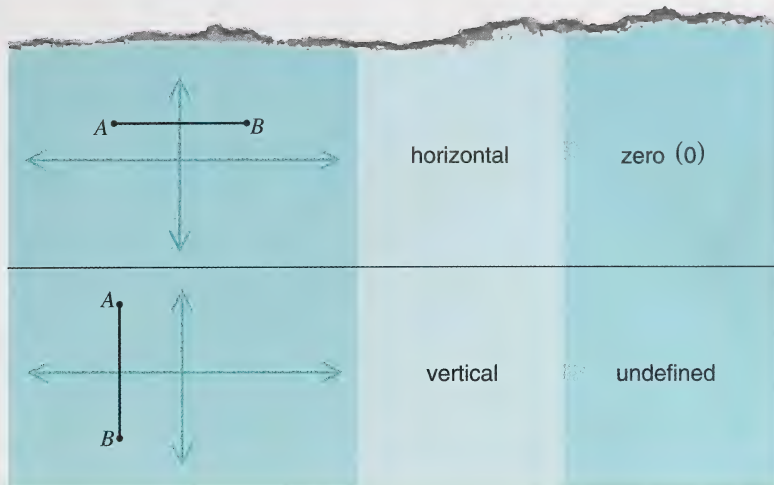
Compare your responses with the suggested answers in the Appendix, Follow-up Activities, pages 70–73.

If you had difficulties understanding the skills and concepts in Module 5, Line Segments, it is recommended that you do the Extra Help. If you have a clear understanding of the skills and concepts in this module, it is recommended that you do the Enrichment. You may decide to do both.

EXTRA HELP

The following table summarizes what you have discovered about the slopes of line segments on the coordinate plane.

Example	Description	Slope
	rises to the right	positive (+)
	falls to the right	negative (–)



When you are calculating the slope of a line segment on the coordinate plane, you should recall this chart and think about the reasonableness of your answer.

1.
 - a. Given $A(-2, 3)$ and $B(4, 3)$, draw the graph of line segment AB .
 - b. What is the slope of line segment AB ?
 - c. Make a general statement about the slope of any line segment parallel to the x -axis.
2.
 - a. Given $P(4, -2)$ and $Q(4, 4)$, draw a graph of line segment PQ .
 - b. What is the slope of line segment PQ ?
 - c. Make a general statement about the slope of any line segment parallel to the y -axis.
3. The vertices of a figure are $P(-2, 0)$, $Q(6, 5)$, $R(1, -2)$, and $S(-7, -7)$.
 - a. Plot the given points on a coordinate plane; then, connect P to Q , Q to R , R to S , and S to P .
 - b. Calculate the slope of each side of the figure.
 - c. What do you notice about the slopes of the sides?
 - d. What type of polygon is figure $PQRS$?

Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Extra Help, pages 74–77.

ENRICHMENT

If you completed the Enrichment in the Follow-up Activities to Module 1: Measurement, you know that you can link your TI-83 graphing calculator to another calculator and/or to a computer and transfer programs. In Module 1: Measurement, you downloaded the CONVERT program from the Addison Wesley website to your graphing calculator, and you used this program to make measurement conversions.

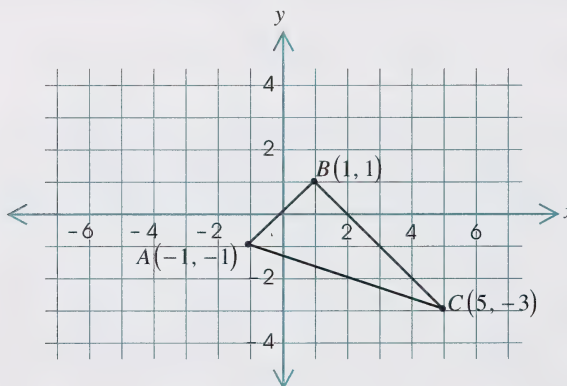
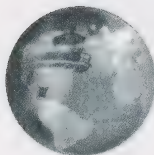
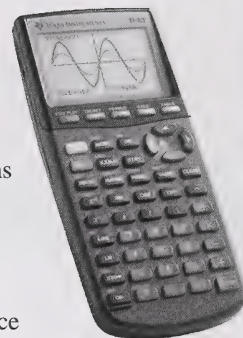
In this module, you use the LINK feature to download programs for finding the distance between two points on the coordinate plane, the midpoint of a line segment, and the slope of a line segment.

Visit the Texas Instruments website; Texas Instruments maintains an archive of programs for their calculators:

<http://www.ticalc.org>

Use the search engine on this site to find programs for the distance formula, midpoint, and slope. Instructions are also given for unzipping and downloading programs.

After you have successfully transferred the programs to your graphing calculator, use the given graph and your graphing calculator to answer the following questions. (Round answers to one decimal place.)



1. What is the length of AB ? BC ? AC ?
2. What is the midpoint of AB ? BC ? AC ?
3. What is the slope of AB ? BC ? AC ?

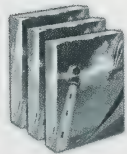
Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Enrichment, pages 77–80.



NATHAN BENN/CORBIS

Completing the Project

By now you should have completed your initial research on water rides. Now you are ready to determine what length and slope of a downhill section of a water ride will allow a rider to make it safely up the next uphill section of the ride.

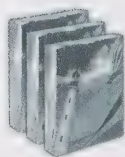


1. Turn to pages 230 and 231 of the textbook and review “Investigating Water Rides.” Answer the activities in the bullets. **Note:** Be sure to make the graph described in the last bullet on page 231.

Keep the data you collect and the responses to the exercises in the project section of your mathematics binder. You will need this information to complete the work in the Module 5 Project Booklet.

2. What did you discover about the distance the marble will travel uphill?

Compare your responses with the suggested answers in the Appendix, Module Project, pages 80–81.



In the final phase of the project, you will design **three** water rides of various thrill levels. You will be expected to make a sketch of your water-ride design and draw a side view of your design on graph paper.

3. To gain more insight into the next phase of this project, turn to page 252 of the textbook and answer exercise 15 of Part C of “What Should I Be Able To Do?”

Compare your response with the suggested answer in the Appendix, Module Project, page 82.



DAVID CUMMING; EYE UBIQUITOUS/CORBIS

Water rides combine the ideas of a water slide and a roller coaster; therefore, you may find it helpful to visit the following sites on the Internet:

- <http://www.discovery.com/exp/rollercoasters/build.html>
- <http://www.learner.org/exhibits/parkphysics/coaster/>

Each of these websites has an activity where you are challenged to design a roller coaster and receive feedback afterwards.

Module Project

Now that you have more insight into the module project, take out the Project Booklet and complete the Module 5 project, Designing a Water Ride.

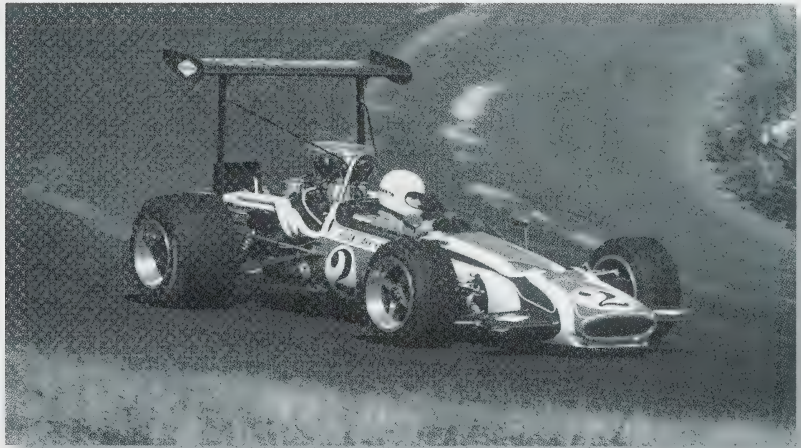
Submit your completed Module 5 Project Booklet to your teacher.

Module Summary

In this module, you explored line segments. You worked with the coordinate system and developed formulas to find the lengths and the midpoints of line segments, given their endpoints. You developed a formula to find the slopes of line segments that have been plotted on a grid.

From your work in this module, you became aware that engineers who design structures, such as cycling tracks and amusement rides, use mathematics in their careers.

Cycling tracks in velodromes are banked, and cyclists use the slope of the track to help them negotiate the turns safely.

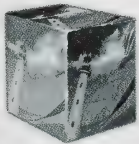


Do you think auto-racing tracks are banked? If so, how will the slope of the banks of an auto-racing track differ from the slope of a cycling track?

Module Assignment

To demonstrate what you have learned in this module, complete the module assignment in the Assignment Booklet.

**Submit your completed Module 5 Assignment Booklet
to your teacher.**





Applied Mathematics 10

APPENDIX

GLOSSARY

Collinear: lying on the same line segment

Oblique: at a slant

Pitch: the vertical rise of a roof in inches for every horizontal run of 12 inches

Rise: the vertical distance between two points on a coordinate plane

Run: the horizontal distance between two points on a coordinate plane

Slope: a numerical measure of the steepness of a line segment; the ratio of the rise to the run

SUGGESTED ANSWERS

Activity 1: Distance

1. a. $53^\circ - 44^\circ = 9^\circ$

Edmonton is 9° latitude farther north than Halifax.

b. $123^\circ - 75^\circ = 48^\circ$

Victoria is 48° longitude farther west than Ottawa.

2. a. The vertical line segment AC is 5 units long.

b. Subtract -2 from 3.

$$\begin{aligned} 3 - (-2) &= 3 + 2 \\ &= 5 \end{aligned}$$

c. Given two endpoints (x_1, y_1) and (x_2, y_2) , the length of the vertical line segment is either $y_1 - y_2$ or $y_2 - y_1$, whichever is positive. **Note:** The subscripts are used to distinguish the x - and y -coordinates of the two points.

3. a. The horizontal line segment BC is 7 units long.

b. Subtract -3 from 4.

$$\begin{aligned} 4 - (-3) &= 4 + 3 \\ &= 7 \end{aligned}$$

- c. Given two endpoints (x_1, y_1) and (x_2, y_2) , the length of the horizontal line segment is either $x_1 - x_2$ or $x_2 - x_1$, whichever is positive.

4. Textbook exercises 4 and 5 of “Investigation 2: The Distance between Two Points,” p. 203

4. $AB^2 = AC^2 + CB^2$

5. $AB^2 = AC^2 + CB^2$

$$AB^2 = 5^2 + 7^2$$

$$AB^2 = 25 + 49$$

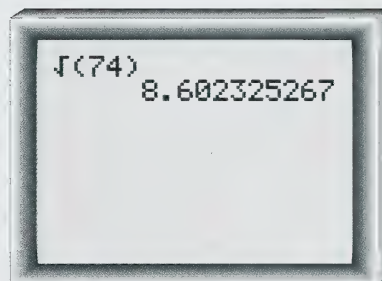
$$AB^2 = 74$$

$$AB = \sqrt{74}$$

$$AB \doteq 8.6$$

The line segment AB is about 8.6 units long.

2nd [$\sqrt{}$] 7 4) ENTER



5. Textbook exercises 3 and 4 of “Discussing the Ideas,” p. 207

3. No, it doesn't matter which x -coordinate is subtracted from which. Since the difference is squared, the order doesn't matter. However, by convention, the first x -coordinate is subtracted from the second x -coordinate. The same is true for the y -coordinates.
4. It is appropriate to use the positive square root; distances generally are positive, not negative.

6. Textbook exercises 3.a., 4, 5, 6, and 7 of “Exercises: Checking Your Skills,” pp. 208 to 210

3. a. The ordered pairs are $(8, 3)$ and $(2, 9)$. Therefore, $x_1 = 8$, $y_1 = 3$, $x_2 = 2$, and $y_2 = 9$.
Use the distance formula.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2 - 8)^2 + (9 - 3)^2}$$

$$= \sqrt{(-6)^2 + 6^2}$$

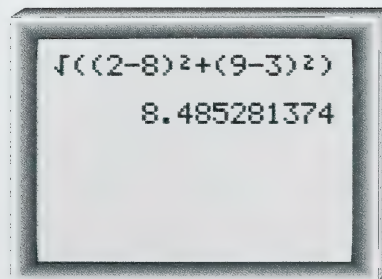
$$= \sqrt{36 + 36}$$

$$= \sqrt{72}$$

$$\doteq 8.5$$

2nd [$\sqrt{}$] (2 - 8) x^2

+ (9 - 3) x^2) ENTER



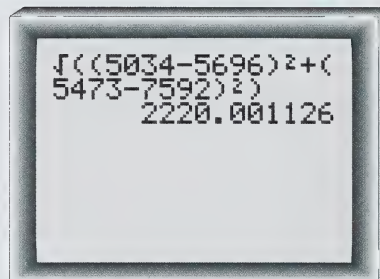
The distance between the flags is about 8.5 units.

Activity 1 (continued)

4. The ordered pairs are (5696, 7592) and (5034, 5473). Therefore,
 $x_1 = 5696$, $y_1 = 7592$, $x_2 = 5034$, and $y_2 = 5473$.

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5034 - 5696)^2 + (5473 - 7592)^2} \\ &= \sqrt{(-662)^2 + (-2119)^2} \\ &\doteq 2220 \end{aligned}$$

Don't let the big numbers scare you!
 The method is exactly the same!

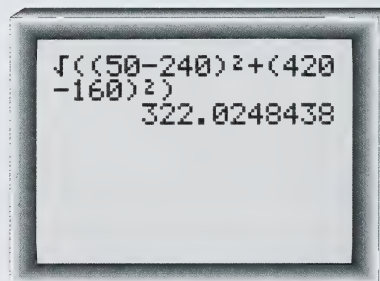


The straight-line distance between Lethbridge and Winnipeg is about 2220 units.

5. a. **Step 1:** Calculate the distance FS , the distance of the second freighter from the ship in distress.
 The endpoints of line segment FS are $F(240, 160)$ and $S(50, 420)$. Therefore,
 $x_1 = 240$, $x_2 = 50$, $y_1 = 160$, and $y_2 = 420$.

$$\begin{aligned} FS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(50 - 240)^2 + (420 - 160)^2} \\ &= \sqrt{(-190)^2 + 260^2} \\ &\doteq 322 \end{aligned}$$

The distance FS is about 322 nautical miles.



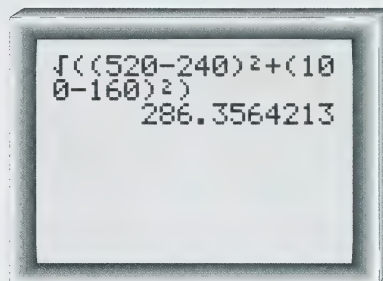
Step 2: Calculate the distance FC , the distance of the coast guard cutter from the ship in distress.

The endpoints of line segment FC are $F(240, 160)$ and $C(520, 100)$. Therefore,

$$x_1 = 240, x_2 = 520, y_1 = 160, \text{ and } y_2 = 100.$$

$$\begin{aligned} FC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(520 - 240)^2 + (100 - 160)^2} \\ &= \sqrt{280^2 + (-60)^2} \\ &\doteq 286 \end{aligned}$$

The distance FC is about 286 nautical miles.



Step 3: Compare the distances.

$$286 < 322$$

The coast guard cutter is closer to the ship in distress.

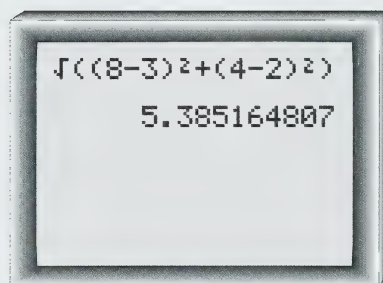
- b. Yes, other factors could be considered. How prepared each ship and crew is to undertake a rescue should be taken into consideration. Weather and tide conditions could be factors. The speed at which each ship could travel may also be a factor. There could be many other factors.

6. Step 1: Determine the endpoints of line segment AB . The endpoints of line segment AB are

$$A(3, 2) \text{ and } B(8, 4).$$

Step 2: Calculate the straight-line distance between A and B .

$$\begin{aligned} AB &= \sqrt{(8-3)^2 + (4-2)^2} \\ &= \sqrt{5^2 + 2^2} \\ &= \sqrt{25+4} \\ &= \sqrt{29} \\ &\doteq 5.4 \end{aligned}$$



The straight-line distance is about 5.4 units.

Step 3: Use logic to decide if the straight-line distance is the actual distance you would walk.

No, this is not the actual distance you would walk. The photograph shows many topographical features. You would probably travel on roads, walkways, and paths. You would go around buildings and other obstructions. You would also traverse natural obstacles, such as hills, creeks, and brush.

Activity 1 (continued)

7. **Step 1:** Estimate the length of the water ride. Estimates will vary.

The graph is almost a straight line that covers about 100 units.

- Step 2:** Calculate the length of the first section, from $(10, 8)$ to $(20, 4)$.

$$\begin{aligned}\text{length from } (10, 8) \text{ to } (20, 4) &= \sqrt{(20-10)^2 + (4-8)^2} \\ &= \sqrt{10^2 + (-4)^2} \\ &= \sqrt{100 + 16} \\ &= \sqrt{116} \\ &\doteq 11\end{aligned}$$

The first section is about 11 units in length.

- Step 3:** Calculate the length of the second section, from $(20, 4)$ to $(40, 6)$.

$$\begin{aligned}\text{length from } (20, 4) \text{ to } (40, 6) &= \sqrt{(40-20)^2 + (6-4)^2} \\ &= \sqrt{20^2 + 2^2} \\ &= \sqrt{400 + 4} \\ &= \sqrt{404} \\ &\doteq 20\end{aligned}$$

The second section is about 20 units in length.

- Step 4:** Calculate the length of the third section, from $(40, 6)$ to $(70, 3)$.

$$\begin{aligned}\text{length from } (40, 6) \text{ to } (70, 3) &= \sqrt{(70-40)^2 + (3-6)^2} \\ &= \sqrt{30^2 + (-3)^2} \\ &= \sqrt{900 + 9} \\ &= \sqrt{909} \\ &\doteq 30\end{aligned}$$

The third section is about 30 units in length.

Step 5: Calculate the length of the fourth section, from $(70, 3)$ to $(80, 2)$.

$$\begin{aligned}\text{length from } (70, 3) \text{ to } (80, 2) &= \sqrt{(80-70)^2 + (2-3)^2} \\ &= \sqrt{10^2 + (-1)^2} \\ &= \sqrt{100+1} \\ &= \sqrt{101} \\ &\doteq 10\end{aligned}$$

The fourth section is about 10 units in length.

Step 6: Calculate the length of the fifth section, from $(80, 2)$ to $(110, 1)$.

$$\begin{aligned}\text{length from } (80, 2) \text{ to } (110, 1) &= \sqrt{(110-80)^2 + (1-2)^2} \\ &= \sqrt{30^2 + (-1)^2} \\ &= \sqrt{900+1} \\ &= \sqrt{901} \\ &\doteq 30\end{aligned}$$

The fifth section is about 30 units in length.

Step 7: Calculate the total length. Add the lengths in Steps 2 to 7.

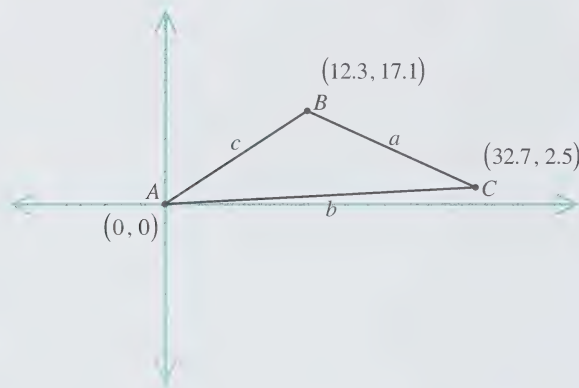
$$\begin{aligned}\text{total length} &\doteq 11 + 20 + 30 + 10 + 30 \\ &\doteq 101\end{aligned}$$

The total length is about 101 units.

Activity 1 (continued)

7. Textbook exercise 8 of “Exercises: Extending Your Thinking,” p. 210

8. The following sketch will be helpful for exercises 8.a. and 8.b.



The side opposite $\angle A$ is a , the side opposite $\angle B$ is b , and the side opposite $\angle C$ is c .
 $\therefore a = BC$, $b = AC$,
 and $c = AB$

a. Step 1: Find a . **Note:** Point A is $(0, 0)$, point B is $(12.3, 17.1)$, and $a = AB$.

$$\begin{aligned} a &= \sqrt{(12.3 - 0)^2 + (17.1 - 0)^2} \\ &= \sqrt{(12.3)^2 + (17.1)^2} \\ &\doteq 21.1 \end{aligned}$$

The distance between A and B is about 21.1 km.

Step 2: Find b . **Note:** Point B is $(12.3, 17.1)$, point C is $(32.7, 2.5)$, and $b = BC$.

$$\begin{aligned} b &= \sqrt{(32.7 - 12.3)^2 + (2.5 - 17.1)^2} \\ &\doteq 25.1 \end{aligned}$$

The distance between B and C is about 25.1 km.

$$\sqrt{(12.3-0)^2 + (17.1-0)^2} = 21.06418762$$

$$\sqrt{(32.7-12.3)^2 + (2.5-17.1)^2} = 25.08625121$$

Step 3: Find c . **Note:** Point A is $(0, 0)$, point C is

$(32.7, 2.5)$, and $c = AC$.

$$\begin{aligned} c &= \sqrt{(32.7-0)^2 + (2.5-0)^2} \\ &= \sqrt{(32.7)^2 + (2.5)^2} \\ &\doteq 32.8 \end{aligned}$$

A calculator screen showing the calculation of the distance between points A(0,0) and C(32.7, 2.5). The input is $\sqrt{(32.7-0)^2 + (2.5-0)^2}$ and the result is 32.79542651.

The distance between A and C is about 32.8 km.

b. Step 1: Find the semi-perimeter. **Note:** Side $a \doteq 25.1$, $b \doteq 32.8$, and $c \doteq 21.1$.

$$\begin{aligned} s &= \frac{1}{2}(a+b+c) \\ &\doteq \frac{1}{2}(25.1+32.8+21.1) \\ &\doteq 39.5 \end{aligned}$$

Step 2: Find the area using Heron's formula.

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &\doteq \sqrt{39.5(39.5-25.1)(39.5-32.8)(39.5-21.1)} \\ &\doteq 264.8 \end{aligned}$$

The area is about 264.8 km^2 .

A calculator screen showing the calculation of the area of a triangle with sides 25.1, 32.8, and 21.1. The input is $\sqrt{39.5(39.5-25.1)(39.5-32.8)(39.5-21.1)}$ and the result is 264.8049546.

8. Textbook exercise "Communicating the Ideas," p. 212

Answers will vary. You may choose to write the distance formula in steps or with different subscripts. You may have chosen to draw a diagram.

Activity 2: Midpoints

1. Textbook exercise 2 of “Discussing the Ideas,” p. 242

2. A midpoint is the point that divides a line segment into two equal parts. A line segment has two endpoints. The x -coordinate of the midpoint is the average of the x -coordinates of the two endpoints. The y -coordinate of the midpoint is the average of the y -coordinates of the two endpoints.

$$\begin{array}{ll} \text{2. a. } x_M = \frac{3+0}{2} & y_M = \frac{4+5}{2} \\ & = 1.5 \qquad \qquad = 4.5 \end{array}$$

The midpoint is $(1.5, 4.5)$.

$$\begin{array}{ll} \text{b. } x_M = \frac{6+9}{2} & y_M = \frac{5+3}{2} \\ & = 7.5 \qquad \qquad = 4 \end{array}$$

The midpoint is $(7.5, 4)$.

$$\begin{array}{ll} \text{c. } x_M = \frac{-2+6}{2} & y_M = \frac{3+(-1)}{2} \\ & = 2 \qquad \qquad = 1 \end{array}$$

The midpoint is $(2, 1)$.

$$\begin{array}{ll} \text{d. } x_M = \frac{(-2)+(-1)}{2} & y_M = \frac{4+0}{2} \\ & = -1.5 \qquad \qquad = 2 \end{array}$$

The midpoint is $(-1.5, 2)$.

$$\begin{array}{ll} \text{3. } x_M = \frac{0+3}{2} & y_M = \frac{3+(-4)}{2} \\ & = 1.5 \qquad \qquad = -0.5 \end{array}$$

The centre of the circle is $(1.5, -0.5)$.

4. Textbook exercises 3 and 4 of "Exercises: Checking Your Skills," p. 244

3. **Step 1:** Find the midpoint between Sunup and Sundown.

$$\begin{aligned}x_M &= \frac{6.3+4.7}{2} & y_M &= \frac{2.9+13.2}{2} \\&= 5.5 & &= 8.05\end{aligned}$$

The midpoint is $(5.5, 8.05)$.

Step 2: Find the distance from Sundown to the midpoint. Use the distance formula.

Sundown is at $(6.3, 2.9)$; the midpoint is at $(5.5, 8.05)$.

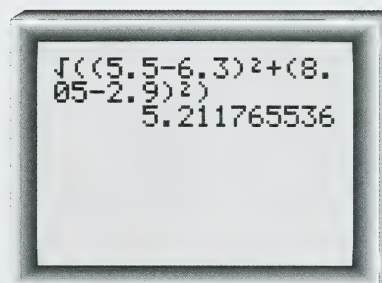
$$\begin{aligned}\text{distance} &= \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} \\&= \sqrt{(5.5 - 6.3)^2 + (8.05 - 2.9)^2} \\&= \sqrt{(-0.8)^2 + 5.15^2} \\&\doteq 5.2\end{aligned}$$

Sundown is about 5.2 km from midpoint.

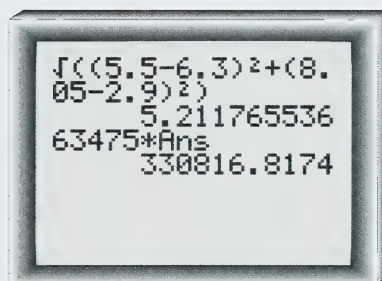
Step 3: Find the cost of the construction.

$$\$63\,475 \times 5.2 \doteq \$330\,816.82$$

Sundown's costs will be about \$330 816.82.



$\sqrt{(5.5-6.3)^2 + (8.05-2.9)^2}$
5.211765536



$\sqrt{(5.5-6.3)^2 + (8.05-2.9)^2}$
5.211765536
63475*Ans
330816.8174

Activity 2 (continued)

4. Step 1: Determine the endpoints of each section.

The endpoints of the first section are $(0, 400)$ and $(1000, 600)$.

The endpoints of the second section are $(1000, 600)$ and $(2000, 1000)$.

The endpoints of the third section are $(2000, 1000)$ and $(5000, 600)$.

The endpoints of the fourth section are $(5000, 600)$ and $(7000, 400)$.

Step 2: Determine the midpoint of each section.

$$\begin{aligned}x_M &= \frac{0 + 1000}{2} & y_M &= \frac{400 + 600}{2} \\ &= 500 & &= 500\end{aligned}$$

The midpoint of the first section is $(500, 500)$.

$$\begin{aligned}x_M &= \frac{1000 + 2000}{2} & y_M &= \frac{600 + 1000}{2} \\ &= 1500 & &= 800\end{aligned}$$

The midpoint of the second section is $(1500, 800)$.

$$\begin{aligned}x_M &= \frac{2000 + 5000}{2} & y_M &= \frac{1000 + 600}{2} \\ &= 3500 & &= 800\end{aligned}$$

The midpoint of the third section is $(3500, 800)$.

$$\begin{aligned}x_M &= \frac{5000 + 7000}{2} & y_M &= \frac{600 + 400}{2} \\ &= 6000 & &= 500\end{aligned}$$

The midpoint of the fourth section is $(6000, 500)$.

5. Textbook exercise “Communicating the Ideas,” p. 245

Answers will vary. Following is an acceptable response.

The halfway point or middle of a line segment is the point whose x -coordinate is the average of the x -coordinates of the two endpoints, and whose y -coordinate is the average of the y -coordinates of the two endpoints.

Activity 3: Slope

1. Textbook exercise 1 of “Exercises: Checking Your Skills,” p. 224

$$\begin{aligned} 1. \quad \text{Slope} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{25}{15} \\ &= \frac{5}{3} \end{aligned}$$

The slope is $\frac{5}{3}$.

$$2. \quad \text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

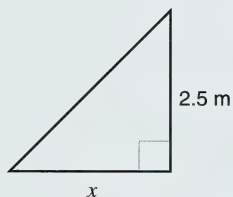
$$4 = \frac{2.5}{x}$$

$$4 \times x = \frac{2.5}{\cancel{x}} \times \cancel{x}^1$$

$$4x = 2.5$$

$$x = \frac{2.5}{4}$$

$$= 0.625$$



The ladder should be placed 0.625 m from the base of the house.

3. The slope is generally steeper in cold regions so snow will not build up on the roof.

$$\begin{aligned} 4. \quad \text{Slope of } AB &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{3}{4} \end{aligned}$$

$A(2, 2)$

$B(6, 5)$

$$\begin{aligned} \text{Slope of } DC &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{-2}{5} \\ &= -\frac{2}{5} \end{aligned}$$

$D(-3, 7)$

$C(2, 5)$

Activity 3: (continued)

$$\begin{aligned}\text{Slope of } EF &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{5}{3}\end{aligned}$$

$$E(0, -5)$$

$$F(3, 0)$$

$$\begin{aligned}\text{Slope of } GH &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{-3}{4} \\ &= -\frac{3}{4}\end{aligned}$$

$$G(-4, 3)$$

$$H(0, 0)$$

$$\begin{aligned}\text{Slope of } JK &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{0}{-4} \\ &= 0\end{aligned}$$

$$J(-1, -2)$$

$$K(-5, -2)$$

$$\begin{aligned}\text{Slope of } LM &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{7}{0} \\ &= \text{undefined}\end{aligned}$$

$$L(5, -5)$$

$$M(5, 2)$$

Division by zero is undefined.

5. Textbook exercises 3 and 4 of “Discussing the Ideas,” p. 224

3. Every horizontal line has a slope of 0 because the rise for a horizontal line will always be 0. In other

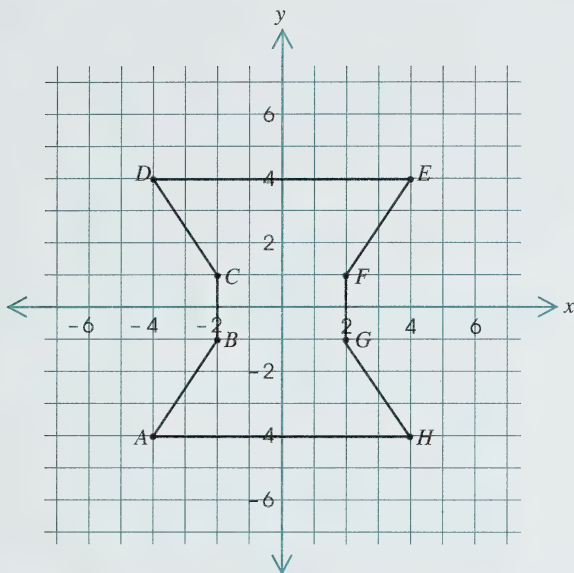
words, in the formula $\frac{y_B - y_A}{x_B - x_A}$, $y_B - y_A = 0$.

4. Every vertical line has an undefined slope because the run for a vertical line will always be 0. In other

words, in the formula $\frac{y_B - y_A}{x_B - x_A}$, $x_B - x_A = 0$. Division by zero is undefined.

6. Textbook exercises 2 and 4 of “Exercises: Checking Your Skills,” pp. 224 and 225

2.



$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\begin{aligned}\text{Slope of } AB &= \frac{-1 - (-4)}{-2 - (-4)} \\ &= \frac{-1 + 4}{-2 + 4} \\ &= \frac{3}{2}\end{aligned}$$

$$\begin{aligned}\text{Slope of } BC &= \frac{1 - (-1)}{-2 - (-2)} \\ &= \frac{1 + 1}{-2 + 2} \\ &= \frac{2}{0} \\ &= \text{undefined}\end{aligned}$$

The slope of a vertical line segment is undefined.

Activity 3 (continued)

$$\begin{aligned}\text{Slope of } CD &= \frac{4-1}{-4-(-2)} \\ &= \frac{3}{-4+2} \\ &= \frac{3}{-2} \\ &= -\frac{3}{2}\end{aligned}$$

$$\begin{aligned}\text{Slope of } DE &= \frac{4-4}{4-(-4)} \\ &= \frac{0}{4+4} \\ &= \frac{0}{8} \\ &= 0\end{aligned}$$

The slope of a horizontal line segment is 0.

$$\begin{aligned}\text{Slope of } EF &= \frac{1-4}{2-4} \\ &= \frac{-3}{-2} \\ &= \frac{3}{2}\end{aligned}$$

$$\begin{aligned}\text{Slope of } FG &= \frac{-1-1}{2-2} \\ &= \frac{-2}{0} \\ &= \text{undefined}\end{aligned}$$

The slope of a vertical line segment is undefined.

$$\begin{aligned}\text{Slope of } GH &= \frac{-4-(-1)}{4-2} \\ &= \frac{-4+1}{2} \\ &= \frac{-3}{2} \\ &= -\frac{3}{2}\end{aligned}$$

$$\begin{aligned}\text{Slope of } HA &= \frac{-4-(-4)}{4-(-4)} \\ &= \frac{-4+4}{4+4} \\ &= \frac{0}{8} \\ &= 0\end{aligned}$$

The slope of a horizontal line segment is 0.

4. $\text{Slope} = \frac{y_B - y_A}{x_B - x_A}$ Use the slope formula.

$$6 = \frac{k-5}{1-2}$$

Substitute known values.

$$6 = \frac{k-5}{-1}$$

$$(-1)6 = \frac{k-5}{-1}(-1)$$

$$-6 = k-5$$

$$-6+5 = k-5+5$$

$$k = -1$$

7. Textbook exercise 10 of “Exercises: Checking Your Skills,” p. 228

10. a. Find the slope of section 1.

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2800 - 2400}{6000 - 0} \\ &= \frac{400}{6000} \\ &= \frac{1}{15}\end{aligned}$$

Find the slope of section 2.

$$\begin{aligned}\text{Slope} &= \frac{2600 - 2800}{16\,000 - 6000} \\ &= \frac{-200}{10\,000} \\ &= -\frac{1}{50}\end{aligned}$$

Find the slope of section 3.

$$\begin{aligned}\text{Slope} &= \frac{2300 - 2600}{22\,000 - 16\,000} \\ &= \frac{-300}{6000} \\ &= -\frac{1}{20}\end{aligned}$$

Find the slope of section 4.

$$\begin{aligned}\text{Slope} &= \frac{2100 - 2300}{30\,000 - 22\,000} \\ &= \frac{-200}{8000} \\ &= -\frac{1}{40}\end{aligned}$$

Activity 3 (continued)

- b. Find the slope of each section as a percent.

$$\begin{aligned}\frac{1}{15} &= \frac{x}{100} \\ 100 \times \frac{1}{15} &= 100 \times \frac{x}{100} \\ \frac{100}{15} &= x \\ x &\doteq 6.7\end{aligned}$$

The first section has a rise of about 6.7%. It would require extra locomotives.

$$\begin{aligned}\frac{1}{50} &= \frac{x}{100} \\ 100 \times \frac{1}{50} &= \frac{x}{100} \times 100 \\ x &= 2\end{aligned}$$

The second section has a rise of 2%. It would not require extra locomotives.

$$\begin{aligned}\frac{1}{20} &= \frac{x}{100} \\ 100 \times \frac{1}{20} &= \frac{x}{100} \times 100 \\ x &= 5\end{aligned}$$

The third section has a rise of 5%. It would require extra locomotives.

$$\begin{aligned}\frac{1}{40} &= \frac{x}{100} \\ 100 \times \frac{1}{40} &= \frac{x}{100} \times 100 \\ x &= 2.5\end{aligned}$$

The fourth section has a rise of 2.5%. It would require extra locomotives.

8. Textbook exercise 11 of "Exercises: Extending Your Thinking," p. 228

11. **Note:** You are to determine only the coordinates of each point.

$$\text{Slope of section 1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-\frac{8}{7} = \frac{y-12}{17-10}$$

$$\frac{-8}{7} = \frac{y-12}{7}$$

$$\frac{-8}{\cancel{7}^1} \times \frac{1}{\cancel{7}_1} = \frac{y-12}{\cancel{7}^1} \times \frac{1}{\cancel{7}_1}$$

$$-8 = y-12$$

$$-8+12 = y-12+12$$

$$y = 4$$

The ordered pair of the end of section 1 is $(17, 4)$.

$$\text{Slope of section 2} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{6}{5} = \frac{y-4}{22-17}$$

$$\frac{6}{5} = \frac{y-4}{5}$$

$$\frac{6}{\cancel{5}^1} \times \frac{1}{\cancel{5}_1} = \frac{y-4}{\cancel{5}^1} \times \frac{1}{\cancel{5}_1}$$

$$6 = y-4$$

$$6+4 = y-4+4$$

$$y = 10$$

The ordered pair of the end of section 2 is $(22, 10)$.

Activity 3 (continued)

$$\text{Slope of section 3} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{-2}{3} = \frac{y - 10}{25 - 22}$$

$$\frac{-2}{3} = \frac{y - 10}{3}$$

$$\frac{-2}{3} \times \frac{1}{3} = \frac{y - 10}{3} \times \frac{1}{3}$$

$$-2 = y - 10$$

$$-2 + 10 = y - 10 + 10$$

$$y = 8$$

The ordered pair of the end of section 3 is $(25, 8)$.

$$\text{Slope of section 4} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{-2}{5} = \frac{y - 8}{30 - 25}$$

$$\frac{-2}{5} = \frac{y - 8}{5}$$

$$\frac{-2}{5} \times \frac{1}{5} = \frac{y - 8}{5} \times \frac{1}{5}$$

$$-2 = y - 8$$

$$-2 + 8 = y - 8 + 8$$

$$y = 6$$

The ordered pair of the end of section 4 is $(30, 6)$.

$$\text{Slope of section 5} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{-4}{1} = \frac{y - 6}{31 - 30}$$

$$\frac{-4}{1} = \frac{y - 6}{1}$$

$$-4 = y - 6$$

$$-4 + 6 = y - 6 + 6$$

$$y = 2$$

The ordered pair of the end of section 5 is $(31, 2)$.

$$\text{Slope of section 6} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{-1}{6} = \frac{y - 2}{37 - 31}$$

$$\frac{-1}{6} = \frac{y - 2}{6}$$

$$\frac{-1}{\cancel{6}_1} \times \frac{1}{\cancel{6}} = \frac{y - 2}{\cancel{6}_1} \times \frac{1}{\cancel{6}}$$

$$-1 = y - 2$$

$$-1 + 2 = y - 2 + 2$$

$$y = 1$$

The ordered pair of the end of section 6 is $(37, 1)$.

Activity 3 (continued)

9. Step 1: Determine the slope of PQ .

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - 2}{-4 - (-1)} \\ &= \frac{4}{-4 + 1} \\ &= \frac{4}{-3} \\ &= -\frac{4}{3}\end{aligned}$$

The slope of PQ is $-\frac{4}{3}$.

Step 2: Determine the slope of QR .

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-2)}{-1 - 2} \\ &= \frac{2 + 2}{-3} \\ &= -\frac{4}{3}\end{aligned}$$

The slope of QR is $-\frac{4}{3}$.

Because the slopes of the line segments joining the points are the same, points P , Q , and R are collinear.

10. Textbook exercise “Communicating the Ideas,” p. 229

Slope is a numerical measure of the steepness of a line segment.

The following formulas are used to determine the slope of a line segment.

- $\text{Slope} = \frac{\text{Rise}}{\text{Run}}$
- $\text{Slope} = \frac{y_B - y_A}{x_B - x_A}$, where (x_A, y_A) and (x_B, y_B) are endpoints of the line segment

Following are the characteristics of line segments with various slopes.

- A line segment that has a positive slope rises to the right.
- A line segment that has a negative slope falls to the right.
- A line segment with a slope of zero is horizontal.
- A line segment with an undefined slope is vertical.

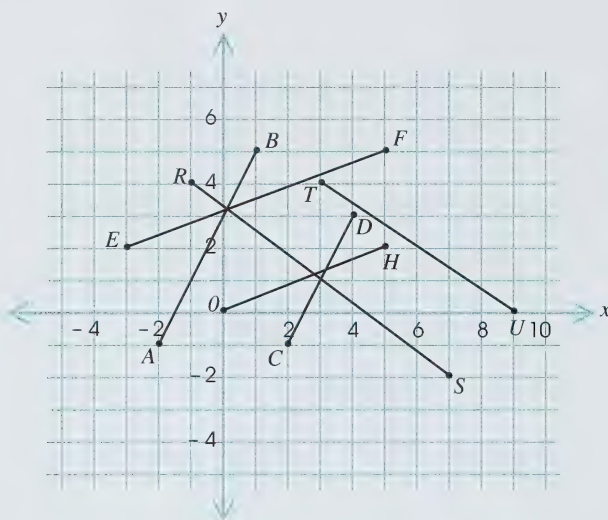
Activity 4: Parallel and Perpendicular Line Segments

1. The slopes of parallel line segments are the same.
2. The slopes of perpendicular line segments are the negative reciprocal of each other; if you multiply the slopes of perpendicular line segments, the product will always be -1 .
3. Textbook exercises 1 to 3 of “Discussing the Ideas,” p. 236
 1. The slopes of parallel line segments are equal. Yes, this is true whether the line segments are vertical, horizontal, rise to the left, or rise to the right.
 2. The product of the slopes of perpendicular line segments is equal to -1 . This is true when one of the line segments rises from left to right, or falls from left to right. This is **not** true when one of the line segments is vertical or horizontal.
 3. Yes, the properties regarding the slopes of line segments also apply to lines.
4. The statement is not true when the line segments are a horizontal line segment and a vertical line segment. The slope of a horizontal line segment is 0. The slope of a vertical line segment is undefined. The product is not -1 .

Activity 4 (continued)

5. Textbook exercises 1 to 3 of “Exercises: Checking Your Skills,” pp. 236 and 237

1. The line segments will look as follows.



$$\begin{aligned}
 \text{a. Slope of } AB &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{5 - (-1)}{1 - (-2)} \\
 &= \frac{5+1}{1+2} \\
 &= \frac{6}{3} \\
 &= 2
 \end{aligned}$$

The slopes are equal.

$$\begin{aligned}
 \text{Slope of } CD &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{3 - (-1)}{4 - 2} \\
 &= \frac{3+1}{2} \\
 &= \frac{4}{2} \\
 &= 2
 \end{aligned}$$

The lines segments are parallel.

$$\begin{aligned}
 \text{b. Slope of } EF &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{5 - 2}{5 - (-3)} \\
 &= \frac{3}{5 + 3} \\
 &= \frac{3}{8}
 \end{aligned}$$

The slopes are not equal.

$$\begin{aligned}
 \text{Slope of } OH &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{2 - 0}{5 - 0} \\
 &= \frac{2}{5}
 \end{aligned}$$

The line segments are not parallel.

$$\begin{aligned}
 \text{c. Slope of } RS &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-2 - 4}{7 - (-1)} \\
 &= \frac{-6}{7 + 1} \\
 &= \frac{-6}{8} \\
 &= \frac{-3}{4} \\
 &= -\frac{3}{4}
 \end{aligned}$$

The slopes are not equal.

$$\begin{aligned}
 \text{Slope of } TU &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{0 - 4}{9 - 3} \\
 &= \frac{-4}{6} \\
 &= \frac{-2}{3} \\
 &= -\frac{2}{3}
 \end{aligned}$$

The line segments are not parallel.

2. Note

Recall that the line segments are perpendicular to each other if the product of their slopes is -1 .

a. $-\frac{3}{2}$

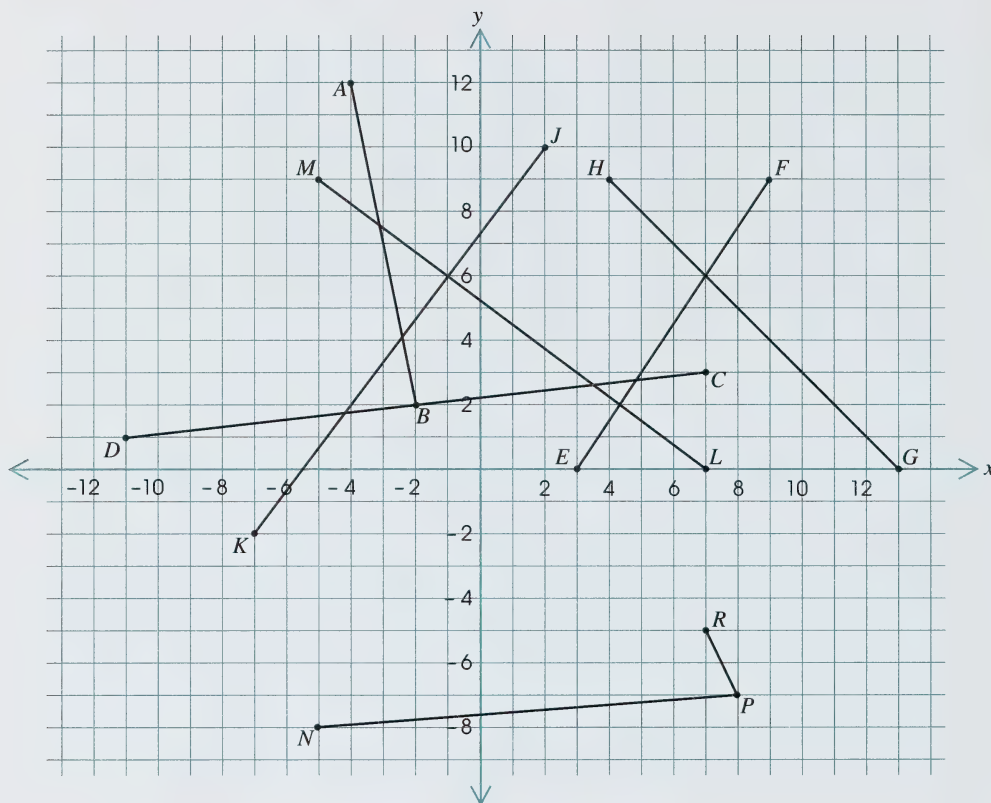
b. $-\frac{8}{5}$

c. 3

d. $\frac{1}{2}$

Activity 4 (continued)

3. The line segments will look as follows.



$$\begin{aligned}
 \text{a. Slope of } AB &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{2 - 12}{-2 - (-4)} \\
 &= \frac{-10}{-2 + 4} \\
 &= \frac{-10}{2} \\
 &= -5
 \end{aligned}$$

$$-5 \times \frac{1}{9} \neq -1$$

$$\begin{aligned}
 \text{Slope of } CD &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{1 - 3}{-11 - 7} \\
 &= \frac{-2}{-18} \\
 &= \frac{1}{9}
 \end{aligned}$$

Line segment AB is not perpendicular to line segment CD .

$$\begin{aligned}
 \text{b. Slope of } EF &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{9 - 0}{9 - 3} \\
 &= \frac{9}{6} \\
 &= \frac{3}{2}
 \end{aligned}$$

$$\frac{3}{2} \times -1 \neq -1$$

$$\begin{aligned}
 \text{Slope of } GH &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{9 - 0}{4 - 13} \\
 &= \frac{9}{-9} \\
 &= -1
 \end{aligned}$$

Line segment EF is not perpendicular to line segment GH .

$$\begin{aligned}
 \text{c. Slope of } JK &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-2 - 10}{-7 - 2} \\
 &= \frac{-12}{-9} \\
 &= \frac{4}{3}
 \end{aligned}$$

$$\frac{4}{3} \times \left(-\frac{3}{4}\right) = -1$$

$$\begin{aligned}
 \text{Slope of } LM &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{9 - 0}{-5 - 7} \\
 &= \frac{9}{-12} \\
 &= -\frac{3}{4}
 \end{aligned}$$

Line segments JK and LM are perpendicular.

$$\begin{aligned}
 \text{d. Slope of } NP &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-7 - (-8)}{8 - (-5)} \\
 &= \frac{-7 + 8}{8 + 5} \\
 &= \frac{1}{13}
 \end{aligned}$$

$$\frac{1}{13} \times (-2) \neq -1$$

$$\begin{aligned}
 \text{Slope of } PR &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-5 - (-7)}{7 - 8} \\
 &= \frac{-5 + 7}{-1} \\
 &= \frac{2}{-1} \\
 &= -2
 \end{aligned}$$

Line segment NP is not perpendicular to PR .

6. Textbook exercise "Communicating the Ideas," p. 237

Answers will vary.

The poster presentation should include the information that the slopes of parallel line segments are equal, and the slopes of perpendicular line segments are the negative reciprocal of each other. **Note:** You may wish to hang your poster in your study area as a study aid.

Follow-up Activities

1. Textbook exercise 8 of Part B of “What Should I Be Able To Do?” p. 250

$$\begin{aligned} 8. \quad \text{Slope} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{4}{10} \quad \leftarrow \text{The run is } \frac{1}{2} \text{ of the base of the triangle.} \\ &= \frac{2}{5} \end{aligned}$$

The slope of a roof is always positive.

The slope is $\frac{2}{5}$.

2. Textbook exercises 9 and 10 of Part B of “What Should I Be Able To Do?” p. 250

$$\begin{aligned} 9. \quad \text{a.} \quad AB &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 4}{2 - (-4)} \\ &= \frac{-6}{2 + 4} \\ &= \frac{-6}{6} \\ &= -1 \end{aligned}$$

The slope of AB is -1 .

$$\begin{aligned} BC &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - (-2)}{4 - 2} \\ &= \frac{6 + 2}{2} \\ &= \frac{8}{2} \\ &= 4 \end{aligned}$$

The slope of BC is 4 .

$$\begin{aligned} AC &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - 4}{4 - (-4)} \\ &= \frac{2}{4 + 4} \\ &= \frac{2}{8} \\ &= \frac{1}{4} \end{aligned}$$

The slope of AC is $\frac{1}{4}$.

$$\begin{aligned}\text{b. } x_M &= \frac{-4+2}{2} \\ &= \frac{-2}{2} \\ &= -1\end{aligned}$$

$$\begin{aligned}y_M &= \frac{4+(-2)}{2} \\ &= \frac{2}{2} \\ &= 1\end{aligned}$$

The midpoint of AB is $(-1, 1)$.

$$\begin{aligned}x_M &= \frac{2+4}{2} \\ &= \frac{6}{2} \\ &= 3\end{aligned}$$

$$\begin{aligned}y_M &= \frac{-2+6}{2} \\ &= \frac{4}{2} \\ &= 2\end{aligned}$$

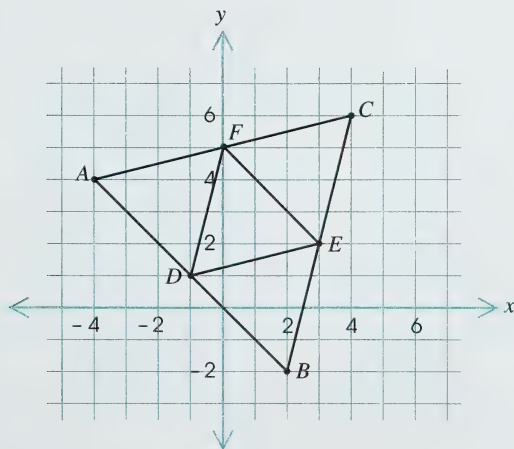
The midpoint of BC is $(3, 2)$.

$$\begin{aligned}x_M &= \frac{-4+4}{2} \\ &= \frac{0}{2} \\ &= 0\end{aligned}$$

$$\begin{aligned}y_M &= \frac{4+6}{2} \\ &= \frac{10}{2} \\ &= 5\end{aligned}$$

The midpoint of AC is $(0, 5)$.

- c. In the given diagram, the triangle formed by joining the midpoints of $\triangle ABC$ has been labelled DEF .



Follow-up Activities (continued)

In 9.a., you determined that the slope of $AB = -1$. The slope of EF , the line segment joining the midpoints of AC and BC , is determined as follows:

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 2}{0 - 3} \\ &= \frac{3}{-3} \\ &= -1\end{aligned}$$

Since the slopes are equal, line segments EF and AB are parallel.

In 9.a., you determined that the slope of BC is 4. The slope of line segment DF , which joins the midpoints of AC and AB , is determined as follows:

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 1}{0 - (-1)} \\ &= \frac{4}{0 + 1} \\ &= \frac{4}{1} \\ &= 4\end{aligned}$$

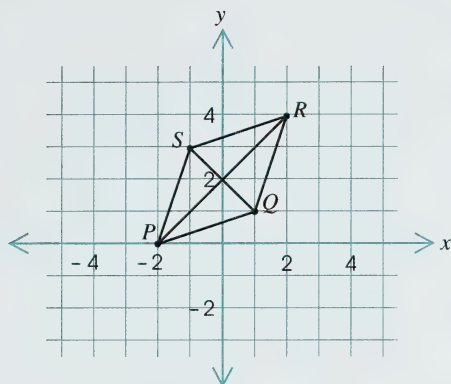
Since the slopes are equal, line segments DF and BC are parallel.

In 9.a., you determined that the slope of AC is $\frac{1}{4}$. The slope of line segment DE , which joins the midpoints of AB and BC , is determined as follows:

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 1}{3 - (-1)} \\ &= \frac{1}{3 + 1} \\ &= \frac{1}{4}\end{aligned}$$

Since the slopes are equal, line segments DE and AC are parallel.

10. You may find it helpful to plot the quadrilateral to illustrate the situation.



Line segments PR and SQ
form the diagonals of the
quadrilateral.

Find the slope of PR .

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 0}{2 - (-2)} \\ &= \frac{4}{2 + 2} \\ &= \frac{4}{4} \\ &= 1\end{aligned}$$

Find the slope of SQ .

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1 - 3}{1 - (-1)} \\ &= \frac{-2}{1 + 1} \\ &= \frac{-2}{2} \\ &= -1\end{aligned}$$

Find the product of the slopes.

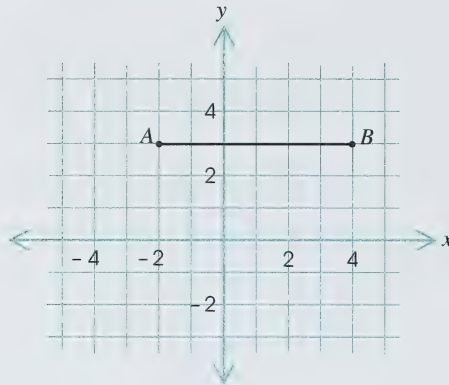
$$1 \times (-1) = -1$$

Since the product of the slopes is -1 , the line segments are perpendicular.

Follow-up Activities (continued)

Extra Help

1. a. The graph of line segment AB will look as follows.

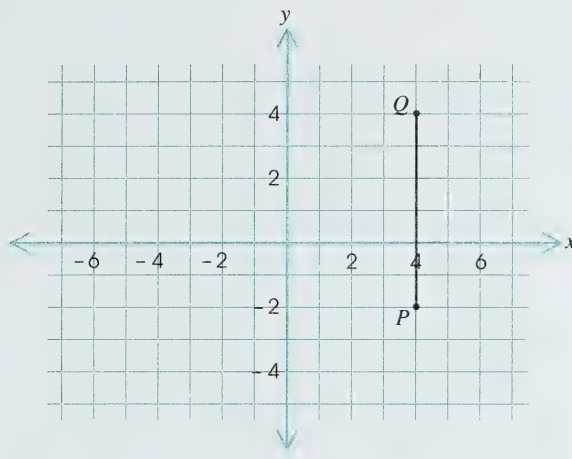


- b. The slope of AB is determined as follows:

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - 3}{4 - (-2)} \\ &= \frac{0}{4 + 2} \\ &= \frac{0}{6} \\ &= 0\end{aligned}$$

- c. The slope of any line segment parallel to the x -axis is 0.

2. a. The graph of line segment PQ will look as follows.



- b. The slope of PQ is determined as follows:

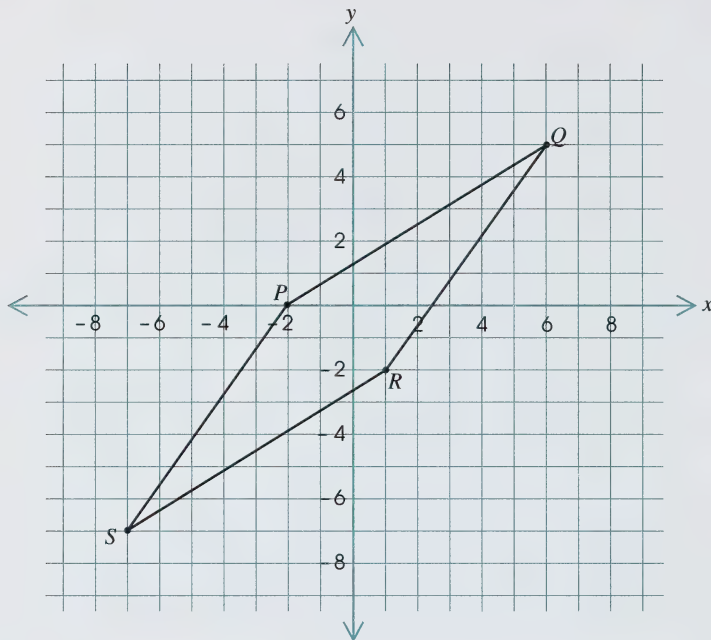
$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - (-2)}{4 - 4} \\ &= \frac{2}{0} \\ &= \text{undefined}\end{aligned}$$

The slope of PQ is undefined.

- c. Since any line segment parallel to the y -axis is also vertical, its slope is undefined.

Follow-up Activities (continued)

3. a. The plot of figure $PQRS$ will look as follows.



- b. The slope of PQ is determined as follows:

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 0}{6 - (-2)} \\ &= \frac{5}{6 + 2} \\ &= \frac{5}{8}\end{aligned}$$

- The slope of RS is determined as follows:

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-7 - (-2)}{-7 - 1} \\ &= \frac{-7 + 2}{-8} \\ &= \frac{-5}{-8} \\ &= \frac{5}{8}\end{aligned}$$

The slope of QR is determined as follows:

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 5}{1 - 6} \\ &= \frac{-7}{-5} \\ &= \frac{7}{5}\end{aligned}$$

The slope of PS is determined as follows:

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-7 - 0}{-7 - (-2)} \\ &= \frac{-7}{-5} \\ &= \frac{7}{5}\end{aligned}$$

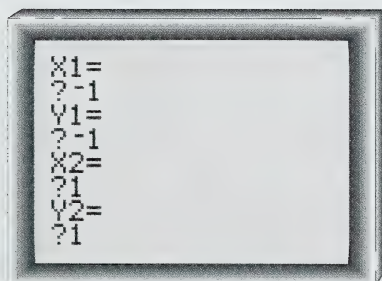
- c. Since the slopes of line segments PQ and RS are equal, the segments are parallel.

Since the slopes of line segments QR and PS are equal, the segments are parallel.

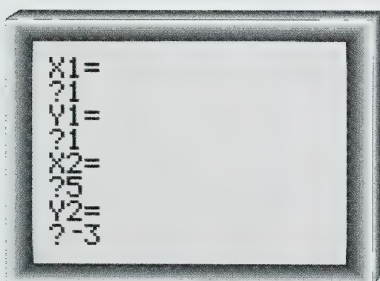
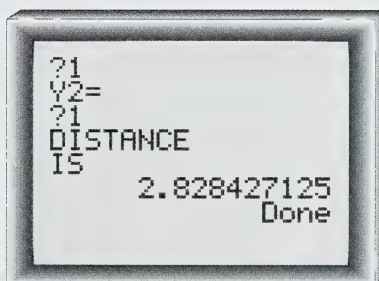
- d. The figure is a parallelogram.

Enrichment

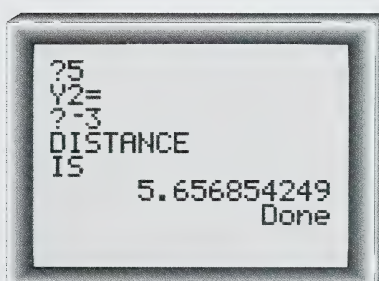
1. Use the distance-formula program to find the length of each line segment.



The length of AB is about 2.8 units.



The length of BC is about 5.7 units.



Follow-up Activities (continued)

$$\begin{matrix} X_1 = 1 \\ Y_1 = 1 \\ X_2 = 1 \\ Y_2 = 3 \end{matrix}$$

```
25
V2=
?-3
DISTANCE
IS
6.32455532
Done
```

The length of AC is about 6.3 units

2. Use the midpoint program to find the midpoint of each line segment.

$$\begin{array}{l} X_1 = \\ ? - 1 \\ Y_1 = \\ ? - 1 \\ X_2 = \\ ?_1 \\ Y_2 = \\ ?_1 \end{array}$$

```

MIDPOINT
IS
X=
Y=
Done

```

The midpoint of AB is $(0, 0)$.

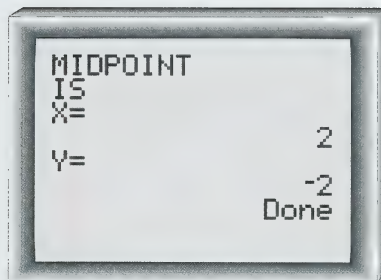
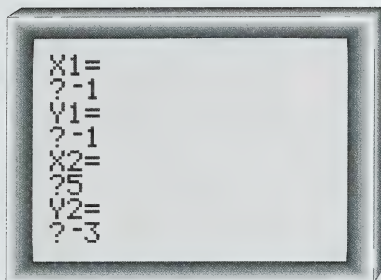
$$\begin{aligned} X_1 &= \\ ?_1 &= \\ Y_1 &= \\ ?_1 &= \\ X_2 &= \\ ?_5 &= \\ Y_2 &= \\ ? &= 3 \end{aligned}$$

```

MIDPOINT
IS
X=
Y=
3
-1
Done

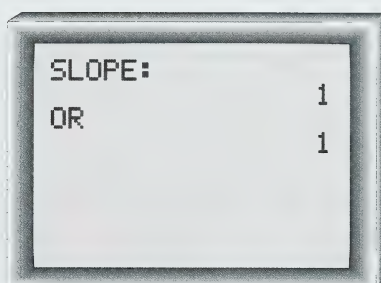
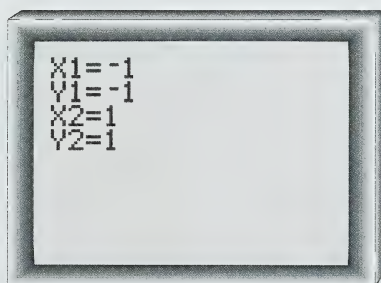
```

The midpoint of BC is $(3, -1)$.

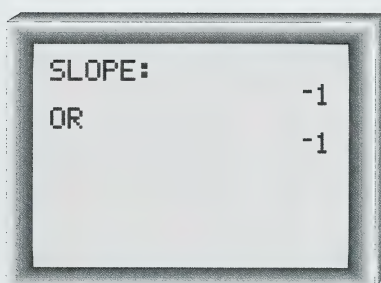
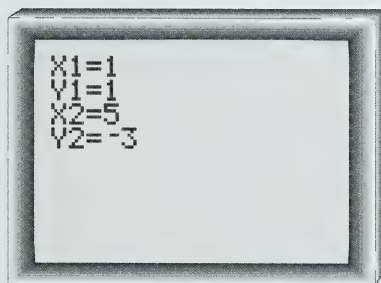


The midpoint of AC is $(2, -2)$.

3. Use the slope program to find the slope of each line segment.

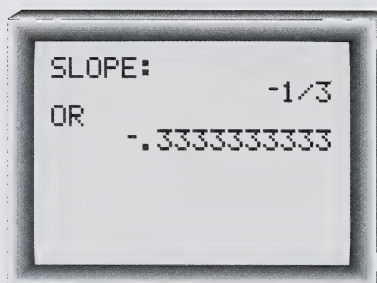
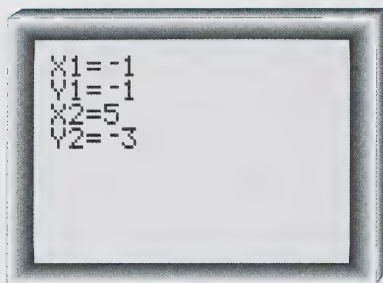


The slope of AB is 1.



The slope of BC is -1 .

Follow-up Activities (continued)



The slope of AC is $-\frac{1}{3}$.

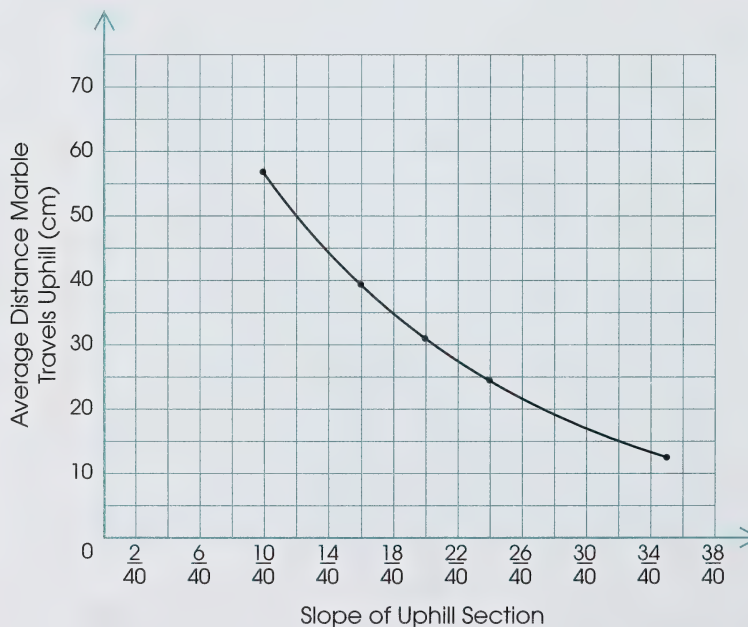
Module Project: Designing a Water Ride

Completing the Project

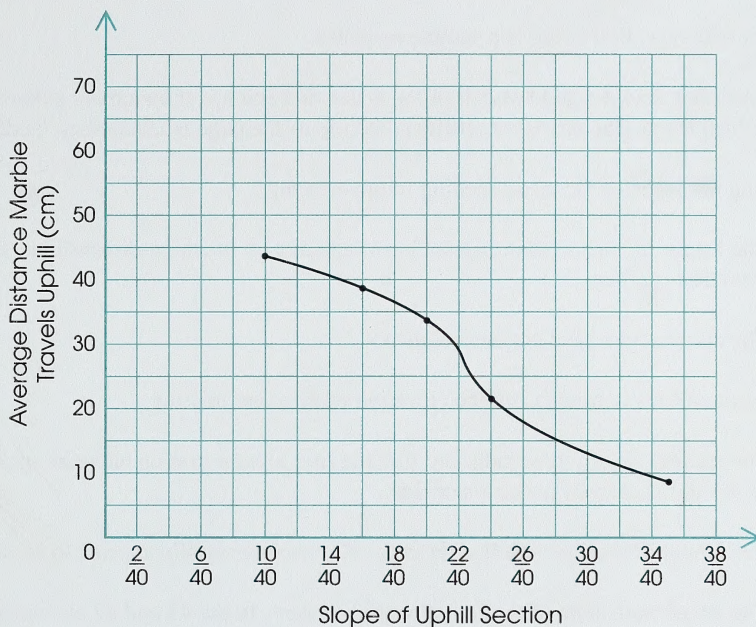
1. Textbook activities in “Investigating Water Rides,” pp. 230 and 231

Answers will vary. The data in exercise 14 on page 251 of the textbook has been used to complete the following graphs as sample responses.

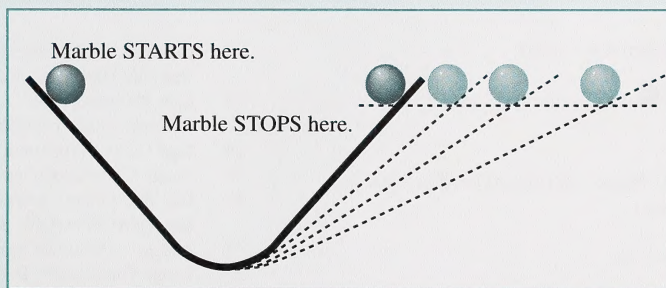
The graph displays the average distance a marble travels uphill when it travels 50 cm downhill on a slope of $\frac{1}{2}$.



The following graph displays the average distance a marble travels uphill when it travels 50 cm downhill on a slope of $\frac{2}{5}$.



2. You should have discovered that the marble will keep rolling until it reaches almost the same height it started from. Without friction, the marble will keep rolling until it reaches the same height (as shown in the diagram).



You should also have discovered that the distance the marble travels up the uphill section is dependent on the slope of the uphill section. If the slope of the uphill section is decreased, the marble will travel farther.

Module Project (continued)

3. Textbook exercise 15 of Part C of “What Should I Be Able To Do?” p. 252

15. Answers will vary. Following is a sample response.

The student has drawn a good sketch of the water ride and has shown three side-elevation profiles of various thrill levels (the elevation profile at the top of the page is labelled as “slide #3”).

Following are some of the areas needing improvement:

- The length of each section has not been included on the three-dimensional sketch or the elevation profiles.
- The calculations have not been shown.
- Some of the coordinates on the elevation profiles are missing.
- The student should have indicated that the first section in each elevation profile is the height of the steps, not part of the water ride.
- The length of the first uphill slide in the #2 elevation profile appears to be too long.
- The length and slope of the first downhill sections in the #1 and #2 elevations do not appear to match the three-dimensional sketch of the slides.

CREDITS

Some clip art drawings are commercially owned.

Welcome Page: Image Club/StudioGear/EyeWire, Inc.

Page

7	(Collage) upper right: Image Club/StudioGear/EyeWire, Inc. lower left: PhotoDisc, Inc.	22	top: Image Club/StudioGear/EyeWire, Inc. bottom: PhotoDisc, Inc.
9	PhotoDisc, Inc.	23	top: PhotoDisc, Inc. bottom: Image Club/StudioGear/EyeWire, Inc.
12–13	bottom right: PhotoDisc, Inc.	24	top: Corel Corporation
15	EyeWire, Inc.	26	Image Club/StudioGear/EyeWire, Inc.
16	PhotoDisc, Inc.	29	top left: Image Club/StudioGear/EyeWire, Inc. top right: PhotoDisc, Inc.
17	middle: Corel Corporation bottom: Image Club/StudioGear/EyeWire, Inc.	31	Image Club/StudioGear/EyeWire, Inc.
18	Image Club/StudioGear/EyeWire, Inc.	32	Image Club/StudioGear/EyeWire, Inc.
19	top and middle: Image Club/StudioGear/EyeWire, Inc.	33	top: PhotoDisc, Inc. middle: Image Club/StudioGear/EyeWire, Inc.
21	Gazelle Technologies, Inc.	34	Image Club/StudioGear/EyeWire, Inc.
		35	Image Club/StudioGear/EyeWire, Inc.
		41	Corel Corporation
		43	Corel Corporation

